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On the Heuristics of JSM Research (Additions to Articles)[1]

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Abstract—The logical means of detecting empirical regularities using the JSM method of automated research support are considered. Generators of hypotheses about the causes and hypotheses about predictions that are stored in sequences of expandable fact bases are determined. Many "histories of possible worlds" are considered, where "world" refers to an expandable fact base. This set is used to determine empirical regularities, that is, empirical laws, tendencies, and weak tendencies. Empirical regularities are used to determine empirical modalities of necessity (for empirical laws), possibilities (for empirical tendencies), and weak possibilities (for weak empirical tendencies). The Propositional calculi of the class ERA are proposed, that is, modal logics with two empirical modalities of necessity and possibility such that they imitate abductive inference through the axioms of abduction $(\Box(p \to q) \,\&\, Tq) \to \Box p)$, $(\Diamond(p \to q) \,\&\, Tq) \to \Diamond p)$, where $\Box$, $\Diamond$, T are operators of necessity, possibility, and truth ("it is true that..."). A series of definitions related to the characterization of data mining using heuristics of the JSM method of automated research support is given.

Keywords: JSM reasoning, JSM research, JSM method of automated research support, induction, analogy, abduction, empirical regularity, modalities of necessity, possibility and weak possibility, nomological statements

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INTRODUCTION

The JSM method of automated research support (ASSR JSM method) is an instrument of the science of artificial intelligence (AI), covering its three main sections: the presentation of knowledge, automated reasoning and computer realizations in the form of artificial intelligence products (AI systems, intelligent systems and AI-robots).

The ASSR JSM method has two stages of realization: the first (initial) stage is the application of the JSM reasoning to the initial fact base FB(0); the second stage consists of applying JSM reasoning to a sequence of expandable fact bases $FB(0) \subset \ldots \subset FB(s)$, and its purpose is to detect empirical regularities ERs: empirical laws (ELs) and empirical tendencies (ETs and WETs, where WETs are suspicious or weak ETs).

The formalization of the means of detection ER = EL $\cup$ ET $\cup$ SET was presented in [1, 2], where the empirical regularity is understood as the simultaneous **preservation** of hypotheses about the causes of the investigated effect and the corresponding hypotheses about its ideas, using hypotheses about the reasons. The generation of hypotheses about the causes and hypotheses about predictions is carried out using inductive inference rules and inference rules by analogy, respectively, and is accompanied by a check of the degree of abductive explanation of fact bases $\rho^{(\sigma)}(p)$, where $\sigma = \{+, -\}$, and $p = 0, 1, \ldots, s$ [1, 2].

1. THE LANGUAGES OF THE ASSR JSM METHOD JL, MJL AND JSM REASONING

The formalization of JSM reasoning is carried out using the JL object language [1–3] and the MJL metalanguage, as discussed below.

We note that using MJL the second stage of the ASSR JSM method is formalized, that is, **JSM research** [1, 2], the result of which is the discovery of many empirical regularities (ERs) and the formation and completion of **open** quasi-axiomatic theories (QATs) [4].

The JL Object Language

X, Z, V (which may have lower indices) are variables for objects and subobjects (variables of grade 1);

Y, U, W (which may have lower indices) are variables for effects (many properties), that is, variables of grade 2;

C, C_1, C_2, ... are constants representing objects and subobjects (values of variables of grade 1);

Q, Q_1, Q_2, ... are constants (many properties), which are the values of variables of grade 2;

[1] [1], [2].

n, m, l, k, r, s (which may have lower indices) are variables, the values of which are natural numbers ($n \in N$), that is, variables of grade 3;

$-$ (complement, difference), $\cap$, $\cup$ are operations of the algebra of sets;

$=$ is an equality predicate for terms of grades 1,2,3;

$\geq$, $\leq$ are predicates for terms of grade 3;

$X \Rightarrow_1^{(p)} Y$ is the predicate "The object X has a set of properties Y," where p is a parameter indicating the applicability of the predicate in the fact base FB(p), $p = 0, 1, \ldots, s$;

$V \Rightarrow_2^{(p)} Y$ is the predicate "V is the cause of Y," where p is a similar parameter;

$\neg$, &, $\vee$, $\rightarrow$ are logical connectives of two-valued logic;

$J_{\bar{\nu}}-j$ are B. Rosser, that is, A. Türkett operators [5], where $\bar{\nu} = \langle \nu, n\rangle$ or $\bar{\nu} = (\tau, n)$;

1, -1, 0, τ are types of truth values: "actual truth," "actual false," "actual contradiction" and "uncertainties," respectively;

$\langle \nu, n\rangle$ is the truth value, where n is the likelihood degree of the results of applying the plausible inference rules (induction and analogy [1–3]), which expresses the number of applications of these rules;

(τ, n) is the set of truth values defined recursively: $(\tau, n) = \{\langle 1, n+1\rangle, \langle -1, n+1\rangle, \langle 0, n+1\rangle\} \cup (\tau, n+1)$;

$$J_{\bar{\nu}}\phi = \begin{cases} t, \text{ if } \nu[\varphi] = \bar{\nu} \\ f, \text{ if } \nu[\varphi] \neq \bar{\nu} \end{cases},$$

where $\nu[\varphi]$ is the estimation function φ, and t, f are truth values of two-valued logic ("true," "false");

$\forall, \exists$ are quantifiers of universality and existence for variable of grades 1, 2, 3, respectively.

The terms and formulas of the object language JL are defined in a standard way, but with a significant addition of formulas, terms and quantifiers from tuples of "variable length" [6, 7].

When searching for and discovering empirical regularities in the fact bases (FB) of intelligent systems, it is required to establish the similarity and difference of facts on a finite but previously **indefinite** set of examples. The number of such examples k, therefore, is a variable (k is called the parameter of empirical induction). This circumstance causes the expansion of the language of the logic of predicates of the first order (for models of arbitrary power) by introducing formulas of "variable length" and quantifiers for tuples [7]. JL is a language of weak second-order predicate logic [8], in which a transitive closure is expressible[2], as well, it is a J-definable language of infinite-valued logic with a finite number of types of truth values (1, -1, 0, τ) [10], which corresponds to the four-valued logic of argumentation [11].

The "variable length" formulas in JL are formulas of the form $\exists k \exists X_0 \exists X_1 \ldots \exists X_{k-1} \exists Y_0 \exists Y_1 \ldots \exists Y_{k-1}$ $(\ldots \&_{i=0}^{k-1} J_{\bar{\nu}}(X_i \Rightarrow_r^p Y_i)\ \&\ \ldots), T_1 \cap \ldots \cap T_k = T, \bigvee_{i=1}^{k}(X = X_i) \bigvee_{i=1}^{k}(Y = Y_i)$, where T, T_i are terms.

JSM reasoning is the interaction of inductive inference rules and inference rules by analogy with the subsequent application of accepting the results of plausible conclusions through abduction of the first kind [1, 2, 12].

The plausible inference rules by empirical induction p.i.r.-1 (inductive inference rules) are formalized by M^{σ}-similarity predicates ($\sigma = +, -$) [1, 2, 12, 13]. $M^{\sigma}_{a,n}(V, W)$ are the least predicates of similarity (they are the means of formalizing the First Rule of the inductive derivation of D. S. Mill [14]). $M^{\sigma}_{a,n}(V, W)$ are reinforced by additional conditions $b^{\sigma}, d_0^{\sigma}, d_2^{\sigma}$, where $\sigma = +, -$, and $M^{+}_{x,n}(V, W)$ and $M^{-}_{y,n}(V, W)$ are predicates of ($\pm$)-similarities of examples, representing, respectively, the inductive inference rules with the "prohibition of counterexamples," differences and similarities–differences (the latter two rules are formalization of the corresponding canons of D. S. Mill [14]) [13].

The general form of p.i.r.-1 is

$$(I)^{+}_{x,y}\ \frac{J_{(\tau,n)}(V \Rightarrow_2^{(p)} W), M^{+}_{x,n}(V, W) \& \neg M^{-}_{y,n}(V, W)}{J_{\langle 1,n+1\rangle}(V \Rightarrow_2^{(p)} W)}.$$

Similarly is defined $(I)^{\sigma}_{x,y}$ for $\sigma = -, 0, \tau$, respectively [12, 13]. The pair $\langle x, y\rangle$ is the name of the strategy $Srt_{x,y}$ of the JSM reasoning [6].

We consider a partial case of JSM reasoning such that there is the only possible cause V for the only single-element W. Then, we obtain p.i.r.-1 for $(I)^{\sigma}_{x,y}$ such that there are $J_{(\tau,0)}(V \Rightarrow_2^{(p)} W)$ and $M^{+}_{x,o}(V, W)$, $M^{-}_{y,o}(V, W)$, $J_{\langle \nu,1\rangle}(V \Rightarrow_2^{(p)} W)$, $J_{(\tau,1)}(V \Rightarrow_2^{(p)} W)$, where $\nu = 1, -1, 0$.

Thus, there is one step of inductive inference. The second step of JSM reasoning [6, 12] is the inference by analogy using the plausible inference rule (p.i.r.-2), which uses the consequences of p.i.r.-1.

For the case of JSM reasoning under consideration, the inference by analogy (p.i.r.-2) is formalized by predicates $P^{\sigma}(V, W)$, where $\sigma = +, -, 0, \tau$, and $P^{+}(X, Y) \rightleftharpoons \exists V(J_{\langle 1,1\rangle}(V \Rightarrow_2^{(p)} Y)$[3] & $(V \subset X)$ & $\neg \exists V_0((J_{\langle -1,1\rangle}(V_0 \Rightarrow_2^{(p)} Y) \vee J_{\langle 0,1\rangle}(V_0 \Rightarrow_2^{(p)} Y))\ \&\ (V_0 \subset X)))$. $P^{-}(X, Y)$ is defined similarly.

[2] D.V. Vinogradov in [9] established that, for finite models, JSM rules are expressible in the predicate logic of the first order.

[3] $\rightleftharpoons$ is equality by definition.

For the truth value $\langle 0, 2\rangle$, where "0" is the type of truth value "actual contradiction," $P^0(X, Y)$ is defined as follows:

$P^0(X, Y) \rightleftharpoons \exists V_1 \exists V_2((J_{\langle 1,1\rangle}(V_1 \Rightarrow_2^{(p)} Y)$ & $J_{\langle -1,1\rangle}(V_2 \Rightarrow_2^{(p)} Y)$ & $(V_1 \subset X)$ & $(V_2 \subset X)$ & $\neg(V_1 = V_2)) \vee \exists V_0((J_{\langle 0,1\rangle}(V_0 \Rightarrow_2^{(p)} Y)$ & $(V_0 \subset X))$.

We also define $P^\tau(X, Y) \rightleftharpoons \neg(P^+(X, Y) \vee P^-(X, Y) \vee P^0(X, Y))$ [12].

The inference rules by analogy (p.i.r.-2) are defined as follows:

$$(II)^\sigma_{x,y} \frac{J_{\langle \tau,0\rangle}(X \Rightarrow_1^{(p)} Y), P^\sigma(X,Y)}{J_{\langle \nu,2\rangle}(X \Rightarrow_1^{(p)} Y)},$$

$$\text{where } \nu = \begin{cases} 1, \text{ if } \sigma = + \\ -1, \text{ if } \sigma = -; \\ 0, \text{ if } \sigma = 0 \end{cases}$$

$$(II)^\tau_{x,y} \frac{J_{\langle \tau,0\rangle}(X \Rightarrow_1^{(p)} Y), P^\tau(X,Y)}{J_{\langle \nu,2\rangle}(X \Rightarrow_1^{(p)} Y)}.$$

We note that p.i.r.-1 $(I)^\sigma_{x,y}$ and p.i.r.-2 $(II)^\sigma_{x,y}$, where $\sigma = +, -, 0, \tau$ are defined for JSM reasoning strategies $Str_{x,y}$ [6, 13] and facts bases FB(p), where $p = 0, 1, ..., s$ [1, 2].

The sets of M^σ-predicates and their negations, where $\sigma = +, -$ form, respectively, distributive lattices [13], that is, lattices of the intensionals IntL^σ and $\mathrm{Int}(\neg L^\sigma)$. As for the inductive inference rules p.i.r.-1 $(I)^\sigma$, they correspond to direct products of the intensities of M^σ-predicates and their negations $\neg M^\sigma$ [6, 13]: $IntL^+ \times Int(\neg L^-)$ (corollary $J_{\langle 1,1\rangle}(V \Rightarrow_2^{(p)} Y)$), $Int(\neg L^+) \times InteL^-$ (corollary $J_{\langle -1,1\rangle}(V \Rightarrow_2^{(p)} Y)$), $IntL^+ \times IntL^-$ (corollary $J_{\langle 0,1\rangle}(V \Rightarrow_2^{(p)} Y)$), $Int(\neg L^+) \times Int(\neg L^-)$ (corollary $J_{\langle \tau,1\rangle}(V \Rightarrow_2^{(p)} Y)$). M^σ-predicates and $\neg M^\sigma$-predicates are ordered by the derivability relation.

Extensionals of M^σ-predicates and $\neg M^\sigma$-predicates are defined as binary relations $\{\langle V,Y\rangle \mid M^+_{x,0}(V,Y)\}$, $\{\langle V,Y\rangle \mid M^-_{y,0}(V,Y)\}$, $\{\langle V,Y\rangle \mid \neg M^+_{x,0}(V,Y)\}$, $\{\langle V,Y\rangle \mid \neg M^-_{y,0}(V,Y)\}$.

Accordingly, the extensionals of the direct products of these lattices, which represent the p.i.r.-$1^{(\sigma)}$, are defined [3, 13].

The consistent application of induction (p.i.r.-1) and analogy (p.i.r.-2) for the strategy $Str_{x,y}$ is representable by the operator $O_{x,y}(\Omega(p))$, where $\Omega(p)$ is a set of elementary statements of the form $J_{\bar{\nu}}(C \Rightarrow_1^{(p)} Q), J_{\langle \tau,0\rangle}(C \Rightarrow_1^{(p)} Q)$, where $\bar{\nu} = \langle \nu, 0\rangle$, $\nu = 1, -1, 0$, corresponding to a given fact base $\mathrm{FB}(p) = \{\langle X,Y\rangle \mid J_{\langle 1,0\rangle}(X \Rightarrow_1^{(p)} Y) \vee J_{\langle -1,0\rangle}(X \Rightarrow_1^{(p)} Y) \vee J_{\langle \tau,0\rangle}(X \Rightarrow_1^{(p)} Y)\}$. The set $\Omega(p) = \Omega^+(p) \cup \Omega^-(p) \cup \Omega^\tau(p)$, $p = 0, 1, ..., s$, and it corresponds one-to-one with $\mathrm{FB}(p) = \mathrm{FB}^+(p) \cup \mathrm{FB}^-(p) \cup \mathrm{FB}^\tau(p)$.

Remark 1-1. The set of elementary statements with the type of truth values τ ("uncertain") is given for all extensions of FB(0), that is, $\mathrm{FB}(p) = \mathrm{FB}^+(p) \cup \mathrm{FB}^-(p) \cup \mathrm{FB}^\tau(0)$, where $p = 0, 1, ..., s$, and $\mathrm{FB}(0) \subset \mathrm{FB}(1) \subset ... \subset \mathrm{FB}(s)$. Therefore, we have $\Omega(0) \subset \Omega(1) \subset ... \subset \Omega(s) - s$ extensions of FB(0), which corresponds to s times of application of the JSM reasoning, i.e., the operator $O_{x,y}(\Omega(p))$ and JSM-operator $\bar{O}_{x,y}(\Omega(p)) \rightleftharpoons O_{x,y}(\Omega(p)) \cup \Omega(p)$.

Thus, there is reflexivity and idempotency $\bar{O}_{x,y}(\Omega(p))$:

$1.\ \Omega(p) \subset \bar{O}_{x,y}(\Omega(p)), 2.\ \bar{O}_{x,y}(\bar{O}_{x,y}(\Omega(p)) = \bar{O}_{x,y}(\Omega(p))$.

Remark 2-1. The strategy of JSM reasoning $\mathrm{Str}_{x,y}$ we call the sequential application of inductive inference rules (p.i.r.-1) and inference rules by analogy (p.i.r.-2), which are $(I)^\sigma_{x,y}$ and $(II)^\sigma_{x,y}$, where $\sigma = +, -, 0, \tau$, and every $Str_{x,y}$ has a name $\langle x, y\rangle$ for four direct products of lattices corresponding to the types of truth values $1, -1, 0, \tau$ [6, 13]. The set of all JSM reasoning strategies $Str_{x,y}$ is denoted by $\overline{Str}$.

Remark 3-1. The conditions for the applicability of JSM reasoning are the formalizability of the relationship of similarity of facts, the presence of (+)- and (–)-facts as source data, as well as the existence of implicit "cause–effect" relationships in data arrays that are expressible and definable using the language JL.

In view of the foregoing, the formalization of the ideas of C.S. Pierce on abduction using JSM reasoning uses the existence of a "cause–effect" relationship defined by predicates $X \Rightarrow_1^{(p)} Y$ ("The object X has the effect of Y") and $V \Rightarrow_2^{(p)} Y$ ("The subobject V is the cause of the effect Y").

In [1, 2] it was shown that one understanding of abduction (according to C.S. Pierce) can be formalized as a means of **accepting** hypotheses generated by the JSM reasoning [12]. In [1, 2] this definition of abduction was called abduction of the first kind; in these works a different understanding of abduction as a logical inference (abduction of the second kind) realized in the **JSM research**, which is the second stage of the ASSR JSM method, is also formalized.

Abduction of the first kind, which we define further [6, 12], is a possible formalization of the idea of abduction of C.S. Pierce expressed in a famous text [15]: The surprising fact, C, is observed; however, if A were true, C would be a matter of course. Hence, there is reason to suspect that A is true.

This text by C.S. Pierce was interpreted by a number of authors as a process of **accepting** hypotheses [16, 17]. Understanding abduction as accepting hypotheses by explaining a multitude of facts has received a procedural realization [18]:

(1) * A set of facts D is given,

(2) * there is a set of hypotheses H,

(3) * H explain D

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(4) * All hypotheses h from H are plausible.

Of course, such a scheme of explanatory abduction requires clarification:

1^0. How were H obtained?

2^0. What does it mean that H explain D?

3^0. How are assessments of accepted hypotheses generated?

Answers to questions 1^0–3^0 are formulated using the ASSR JSM method, which performs JSM reasoning as a synthesis of three cognitive procedures: induction, analogy, and abduction [12, 19][4].

It has already been noted that formalization of abduction in the ASSR JSM method uses the conditions of applicability of JSM reasoning, which include the assumption that there are "cause–effect" relationships in the set of initial facts that are specified with respect to the set $\overline{Str}$ of the given strategies Strx,y of JSM reasoning.

This assumption is formalized by the causal completeness axioms CCA$^{(\sigma)}$ and their weakening ($\exists\sigma$), where $\sigma = +, -$ [6, 12], which are formulated using the language JL:

$$\mathrm{CCA}^{(+)} : \forall X \forall Y \exists V (J_{\langle 1,0\rangle}(X \Rightarrow_1^{(p)} Y) \to (J_{\langle 1,1\rangle}(V \Rightarrow_2^{(p)} Y) \,\&\, (V \subset X)));$$

$$\mathrm{CCA}^{(-)} : \forall X \forall Y \exists V (J_{\langle -1,0\rangle}(X \Rightarrow_1^{(p)} Y) \to (J_{\langle -1,1\rangle}(V \Rightarrow_2^{(p)} Y) \,\&\, (V \subset X))).$$

The truth of CCA$^{(\sigma)}$ for FB(p) means that each (σ)-fact has a (σ)-reason, where σ = +, –, and, if $J_{\langle \nu,0\rangle}(C \Rightarrow_1^{(p)} Q) \in \Omega^{\sigma}(p)$, where $\nu = \begin{cases} 1, \text{ if } \sigma = + \\ -1, \text{ if } \sigma = - \end{cases}$, then $\exists V(J_{\langle \nu,1\rangle}(V \Rightarrow_2^{(p)} Q)$.

Obviously, almost for every studied array of facts presented in FB(p) of an intellectual system that performs JSM reasoning, CCA$^{(\sigma)}$ is true. However, the applicability of JSM reasoning means that many facts characterized through the cause–effect relationship are not empty. Consequently, the conditions ($\exists\sigma$) hold, that is, weakening of the ACP (σ), where σ = +, –:

$$(\exists^+) \exists X \exists Y \exists V (J_{\langle 1,0\rangle}(X \Rightarrow_1^{(p)} Y) \,\&\, (J_{\langle 1,1\rangle}(V \Rightarrow_2^{(p)} Y) \,\&\, (V \subset X)),$$

$$(\exists^-) \exists X \exists Y \exists V (J_{\langle -1,0\rangle}(X \Rightarrow_1^{(p)} Y) \,\&\, (J_{\langle -1,1\rangle}(V \Rightarrow_2^{(p)} Y) \,\&\, (V \subset X)).$$

Accordingly, we define $\widetilde{\mathrm{FB}}^+(p) = \{\langle X, Y\rangle \mid \exists V(J_{\langle 1,0\rangle}(X \Rightarrow_1^{(p)} Y) \,\&\, J_{\langle 1,1\rangle}(V \Rightarrow_2^{(p)} Y) \,\&\, (V \subset X)),$ $\widetilde{\mathrm{FB}}^-(p) = \{\langle X, Y\rangle \mid \exists V(J_{\langle -1,0\rangle}(X \Rightarrow_1^{(p)} Y) \,\&\, J_{\langle -1,1\rangle}(V \Rightarrow_2^{(p)} Y) \,\&\, (V \subset X)).$

Obviously, if $\widetilde{\mathrm{FB}}^{\sigma}(p) = \mathrm{FB}^{\sigma}(p)$, then true CCA$^{(\sigma)}$, where σ = +, –; otherwise: $\widetilde{\mathrm{FB}}^{\sigma}(p) \subset \mathrm{FB}^{\sigma}(p)$.

Remark 4-1. We note that for formalizing the **interaction** of p.i.r.-1 (induction) and p.i.r.-2 (analogy), the means of the JL object language are sufficient. However, the definition of the JSM reasoning with the condition of accepting the generated hypotheses using an abductive explanation of FB(p) is possible only with the use of the MJL metalanguage of the JL language, which is "richer" than JL.

MJL contains formulas and terms of JL, as well as proper terms $\Omega(p)$, $\Delta(p)$; $g_2(\langle V, Y\rangle, \Omega(p))$, $g_1(\langle Z, Y\rangle, \Omega(p))$, $(I)^{\sigma}(\Omega(p))$, $(\mathrm{II})^{\sigma}(\Omega(p))$, $\tilde{\Delta}(p), \tilde{\Omega}(p), \bar{O}_{x,y}(\Omega(p))$ and formulas $\bar{O}_{x,y}(\Omega(p)) = \tilde{\Omega}_{x,y}(p)$, $(I)^{\sigma}(\Omega(p)) = \tilde{\Delta}_{x,y}(p)$; $J_{\langle 1,1\rangle}(V \Rightarrow_2^{(p)} Y) \in \tilde{\Delta}_{x,y}(p)$, $J_{\langle \tau,1\rangle}(V \Rightarrow_2^{(p)} Y) \in \tilde{\Delta}_{x,y}(p)$, $J_{\langle 1,2\rangle}(Z \Rightarrow_1^{(p)} Y) \in \tilde{\Omega}^{+}_{x,y}(p)$, $J_{\langle -1,2\rangle}(Z \Rightarrow_1^{(p)} Y) \in \tilde{\Omega}^{-}_{x,y}(p)$, $J_{\langle \tau,2\rangle}(Z \Rightarrow_1^{(p)} Y) \in \tilde{\Omega}^{\tau}_{x,y}(p)$.

We also introduce the terms MJL $\widetilde{\mathrm{FB}}^+(p) \rightleftharpoons \{\langle X, Y\rangle \mid J_{\langle 1,0\rangle}(X \Rightarrow_1^{(p)} Y) \,\&\, \exists V (J_{\langle 1,1\rangle}(V \Rightarrow_2^{(p)} Y) \,\&\, (V \subset X))\}$ and similarly define $\widetilde{\mathrm{FB}}^-(p)$. Next, we define the functions $\rho^{\sigma}(p) = \dfrac{|\widetilde{\mathrm{FB}}^{\sigma}(p)|}{|\mathrm{FB}^{\sigma}(p)|}$, where $|\widetilde{\mathrm{FB}}^{\sigma}(p)|$, $|\mathrm{FB}^{\sigma}(p)|$ are the number of elements of the corresponding sets, and σ = +, –. $\rho^{\sigma}(p)$ is a degree of abductive explanation of FB(p).

Now. for a fixed strategy of JSM reasoning from the set of given strategies $\overline{Str}$ we define a JSM reasoning such that its **objective** is to **reduce** the set of **uncertain** facts $\mathbf{FB^{\tau}(p)}$ for a sequence of expandable FB(p), where p = 0, 1,..., s: FB(0) $\subset$ FB(1) $\subset$ ... $\subset$ FB(s).

[4] We note that there are many attempts to formalize the ideas of C.S. Peirce on abduction by means of logic and programming using deduction [20–22].

***Df.*1-1.** Let be $\bar{\rho}^{\sigma}$ a given threshold such that $0 \le \bar{\rho}^{\sigma} \le 1$, where $\sigma = +, -$, then, the sequence $\bar{O}_{x,y}(\Omega(0)), \rho^+(0), \rho^-(0), \ldots, \bar{O}_{x,y}(\Omega(s)), \rho^+(s), \rho^-(s)$ is called JSM reasoning. JSM reasoning is called **admissible** if $\bar{\rho} \le \rho^{\sigma}(s)$. The admissible JSM reasoning will be called **monotonically non-decreasing**, if $\rho^{\sigma}(0) \le \rho^{\sigma}(1) \le \ldots \le \rho^{\sigma}(s)$, where $\sigma = +, -$.

For a fixed $Str_{x,y}$, we consider the JSM reasoning defined by the JSM operator $\bar{O}_{x,y}(\Omega(p))$ and functions $\rho^{\sigma}(p)$. Let us determine an abduction of the first kind using JSM reasoning, formalizing the **acceptance** of hypotheses generated by inductive inference (p.i.r.-1) and inference by analogy (p.i.r.-2).

The definition of abduction introduced below characterizes two types of abduction of the first kind: **strong** abduction and **weak** abduction.

***Df.*2-1.** Strong abduction scheme: sets are given $\Omega(p) = \Omega^+(p) \cup \Omega^-(p) \cup \Omega^{\tau}(p)$, $\bar{O}_{x,y}(\Omega(p)) = \tilde{\Omega}(p)$, $(I)_{x,y}(\Omega(p)) = \tilde{\Delta}(p)$, where $(I)_{x,y}(\Omega(p)) = \{(I)^+_{x,y}(\Omega(p)), (I)^-_{x,y}(\Omega(p)), (I)^0_{x,y}(\Omega(p)), (I)^{\tau}_{x,y}(\Omega(p))\}$, and $(I)^{\sigma}_{x,y}(\Omega(p)) = \tilde{\Delta}^{\sigma}(p)$, where $\sigma = +, -, 0, \tau$.

(1) Ω(p) are representations of FB(p); (2) $\tilde{\Delta}(p) \cup \tilde{\Omega}(p)$ are JSM reasoning results (according to Df.1-1); (3) CCA$^{(+)}$, ACP$^{(-)}$ are true with respect to FB(p); (4) Then, the hypotheses $\tilde{\Delta}(p) \cup \tilde{\Omega}(p)$ are **accepted**. Conditions (1)*–(3)* and their Corollary (4)* determine strong abduction of the first kind.

<u>**Remark 5-1**</u>. We consider MJL, using the metapredicate Asr(φ) we define "acceptance of the hypothesis φ," where $\varphi \in \tilde{\Delta}(p)$ or $\varphi \in \tilde{\Omega}(p)$.

We note that φ may be a true J-formula, but is **not accepted** if CCA$^{(\sigma)}$ are not true. We obtain the following scheme of strong abduction, which is a means of accepting the results of the JSM reasoning obtained using p.i.r.-1 (induction) and p.i.r.-2 (analogy):

$$\begin{array}{l} (1)\ \Omega(p) \\ (2)\ \tilde{\Delta}(p) \cup \tilde{\Omega}(p) \\ (3)\ \mathrm{CCA}^{(+)},\ \mathrm{CCA}^{(-)} \\ \hline (4)\ \forall\varphi((\varphi \in \tilde{\Omega}(p) \cup \tilde{\Delta}(p)) \to \mathrm{Acp}(\varphi)). \end{array}$$

We note that φ, $\forall\varphi$, Acp(φ) are means of MJL, and Condition (3) means that all the facts from the FB(p) presented in $\Omega(p)$ have an explanation through the generated hypotheses about the causes of the studied effects. It is obvious that $\rho^{\sigma}(p) = 1$ is tantamount to the truth of CCA$^{(\sigma)}$, where $\sigma = +, -$.

We now define weak abduction of the first kind corresponding to truth ($\exists^{\sigma}$), rather than CCA$^{(\sigma)}$, which means **causal incompleteness** of FB(p): not every (σ)-fact has an explanation through the generated hypothesis about the cause of the effect.

***Df.*3-1.** Weak abduction scheme:

$$\begin{array}{l} (1)'\ \Omega(0), \ldots, \Omega(s), \\ \quad \Omega(0) \subset \ldots \subset \Omega(s), \\ (2)'\ \tilde{\Delta}(0), \ldots, \tilde{\Delta}(s); \tilde{\Omega}(0), \ldots, \tilde{\Omega}(s),\ \rho^+(s) \ge \bar{\rho}^+, \\ \rho^-(s) \ge \bar{\rho}^-,\ \text{where } \bar{\rho}^{\sigma} - \text{set thresholds}^4; \\ (3)'\ (\exists^+), (\exists^-) \\ \hline (4)'\ \forall\varphi((\varphi \in \tilde{\Omega}(s) \cup \tilde{\Delta}(s)) \to \mathrm{Acp}(\varphi)). \end{array}$$

We say that there is practical convergence of the JSM reasoning $\bar{O}_{x,y}(\Omega(0)), \rho^+(0), \rho^-(0), \ldots, \bar{O}_{x,y}(\Omega(s)), \rho^+(s), \rho^-(s)$, if the set thresholds $\bar{\rho}^{\sigma}$: $\rho^{\sigma}(s) \ge \bar{\rho}^{\sigma}$, where $\sigma = +, -$, are attainable for FB(0) $\subset \ldots \subset$ FB(s).

If $\rho^{\sigma}(0) \le \ldots \le \rho^{\sigma}(s)$ occurs, we say that **uniform** practical convergence of the JSM reasoning is realized (this turns out to be essential in determining empirical regularities [1, 2]).

Let us formulate the features of formalizing abduction of the first kind as a means of accepting the generated hypotheses in the MJL language below.

1^0. The idea of abduction is refined and formalized using the ASSR JSM method; therefore, the conditions of its applicability are assumed: the existence of (+)- and (–)- facts, as well as the presence in the FB(p) of positive and negative reasons (($\pm$)-reasons), respectively. These assumptions are formalized using the CCA$^{(\sigma)}$ and ($\exists^{\sigma}$), which are **sufficient** grounds for accepting the generated hypotheses through explanation.

2^0. ($\pm$)-facts and ($\pm$)-reasons are arguments and counterarguments when generating hypotheses about causes and hypotheses about predictions, respectively, for induction and analogy.

3^0. Both abduction schemes are **structurally** feasible by induction for $\tilde{\Delta}(p)$ and analogy for $\Omega(p)$.

4^0. The act of acceptance for strong abduction is based on the truth of CCA$^{(\sigma)}$, a causal completeness axiom [12], which is a means of explaining the presence of an effect in FB(p) and is a sufficient basis for accepting the generated hypotheses[6].

5^0. If only ($\exists^{\sigma}$) is satisfied, and not CCA$^{(\sigma)}$, then it is necessary to consider the dynamic expansion of the FB(p), starting with the FB(0), the **control** of which is provided by functions of the degree of abductive explanation $\rho^{\sigma}(p)$, where $p = 0, 1, \ldots, s$, a $\rho^{\sigma}(s) \ge \bar{\rho}^{\sigma}$.

[5] $\bar{\rho}^{\sigma} \le 1$, in recognition problems often get $\bar{\rho}^{\sigma} = 0.8$.
[6] We can assume that CCA$^{(\sigma)}$ is the principle of induction (D.S. Mill in [14] considered the law of uniformity of nature to be such).

Thus, the JSM reasoning is the synthesis (interaction) of induction, analogy, and abduction, and the latter controls the JSM reasoning process and sets its completion if the threshold is reachable: $\rho^{\sigma}(s) \geq \bar{\rho}^{\sigma}$.

6^0. Abduction of the first kind is expressible in MJL and inexpressible in JL, since the statement of acceptance of hypotheses is realized through a meta-predicate Asr(φ). In addition, weak abduction uses functions $\rho^{\sigma}(p)$.

7^0. An essential feature of the formalization of abduction of the first kind is the fact that abduction is applicable not in **closed** theories, but in **open** ones; moreover, abduction in the ASSR JSM method is a means of forming open theories (and their families): quasi-axiomatic theories defined in [2, 4].

8^0. In formalizing abduction of the first kind, a **coherent theory of truth** is used: acceptance of a statement through a given set of consistent statements [23, 24], namely:

(1) **Asr(φ), if and only if** $\mathbf{CCA}^{(\sigma)}$;

(2) **Asr(φ), if and only if** $\boldsymbol{\rho^{\sigma}(s) \geq \bar{\rho}^{\sigma}}$ and $(\exists^{+}), (\exists^{-})$;

respectively, for strong abduction and weak abduction, where $\sigma = +, -$.

We note that Conditions (1) and (2) are different from Condition *T* of the correspondent theory of truth: *x* is true if and only if *p*, where *x* is the name of the statement *p* [25, 26]. (1) and (2) represent the conditions of acceptance φ (i.e. statement Asr(φ)) with meta-predicate Asr), expressed by consistent knowledge $\mathrm{CCA}^{(\sigma)}$ and $\rho^{\sigma}(s) \geq \bar{\rho}^{\sigma}$ (for $\Omega(0) \subset \Omega(1) \subset \ldots \subset \Omega(s)$), respectively.

2. PREDICATES FOR CONSERVATION OF THE HYPOTHESIS AND CAUSAL FORCINGS OF THE RESEARCH EFFECTS

In this section, we preserve the assumptions of §1: we assume that there is an effect and an only cause corresponding to it, such that there is no iteration of the applications of p.i.r.-1 and p.i.r.-2.

In [1, 2], the idea of detecting empirical regularities was considered and their formalizations were proposed. Empirical regularity is understood to mean the **preservation** of the observed effect with the expansion of the multitude of facts representing it [2]. This preservation of the effect consists in the fact that there is a **regularity** of correspondence of the supposed cause and the effect caused by it. The indicated "cause—effect" regularity occurs not only for a given sequence of nested fact bases FB(*p*), $p = 0, 1, \ldots, s$, that is, for $\mathrm{FB}(0) \subset \ldots \subset \mathrm{FB}(s)$, but it (or its modifications) is observed for all possible permutations of elements of the fact bases of this sequence.

We refine the idea of empirical regularities by determining the possible extensions of the fact bases and the possible regularities generated for each extension of FB(*p*). The purpose of considering all possible extensions of the original sequence is to minimize the **randomness of the choice** of extensions of FB(p).

We will use a terminology similar to that adopted for modal logics [27]: FB(*p*) will be called **possible worlds**, and sequences of their extensions FB(0), FB(1), ..., FB(*s*) such that $\mathrm{FB}(0) \subset \mathrm{FB}(1) \subset \ldots \subset \mathrm{FB}(s)$ are **histories of possible worlds**.

Since FB(*p*) is a binary relation, we introduce the following notation: $\mathrm{FB}(p) = R(p)$, $R(1) = R(0) \cup B(1)$, $R(i + 1) = R(i) \cup B(i + 1)$, $i = 0, 1, \ldots, s - 1$; where $R(0) \cap B(i) = \Lambda$, $i = 1, \ldots, s$; $B(i) \cap B(j) = \Lambda$ if $i \neq j$, where Λ is the empty relation.

Thus, we have $R(0) \subset R(1) \subset \ldots \subset R(s)$ for the original $R(0), R(1), \ldots, R(s)$, that is, the "history of the real world" for which we generate $(s + 1)!$ histories of possible worlds (including itself). We note that each history of possible worlds ends with $R(0) \cup B(1) \cup \ldots \cup B(s)$.

We expand MJL and introduce the following notation for the histories of possible worlds

$$HPW_1, \ldots, HPW_j, \ldots, HPW_{(s+1)!}:$$

$$HPW_1 \quad R^1(0), R^1(1), \ldots, R^1(s)$$

$$\ldots \quad \ldots\ldots\ldots\ldots\ldots\ldots$$

$$HPW_j \quad R^j(0),\ R^j(1),\ R^j(s),$$

$$\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots$$

$$HPW_{(s+1)!}, R^{(s+1)!}_{(0)}, R^{(s+1)!}_{(1)}, \ldots, R^{(s+1)!}_{(s)},$$

where $R^1(s) = \ldots = R^j(s) = \ldots = R^{(s+1)!}_{(s)}$.

We denote the set of all histories of possible worlds by $\overline{HPW}$, and the possible worlds (PW) we have already identified by $R^j(i)$, where $i = 0, 1, \ldots, s; j = 1, \ldots, (s + 1)!$.

We also introduce variables for the histories of possible worlds $h, h_1, \ldots$.

Predicates $V \Rightarrow_2^{(p)} Y$ and $Z \Rightarrow_1^{(p)} Y$ are replaced with predicates $H_2(V, Y, p, h)$ and $H_1(Z, Y, p, h)$, respectively.

The second stage of the ASSR JSM method, the detection of empirical regularities, forms **JSM research** [1, 2]. JSM research is set by

(1) HPW_1: $R^1(0), R^1(1), \ldots, R^1(s)$;

(2) $\Omega^{\tau}(0)$,

(3) $\overline{HPW}$,

(4) $\overline{Str}$,

where $\left|\overline{HPW}\right| = (s + 1)!$, and $R^1(0) = \{\langle X, Y\rangle \mid J_{\langle 1,0\rangle} H_1(X, Y, 0, 1) \vee J_{\langle -1,0\rangle} H_1(X, Y, 0, 1) \vee J(\tau,0)\, H_1(X, Y, 0, 1)\}$, and $\Omega_1(0), \Omega_1(1), \ldots, \Omega_1(s)$ one-to-one correspond to $R^1(0), R^1(1), ..., R^1(s)$. A similar correspondence holds for all $R^j(0), ..., R^j(s)$, where $j = 1, ..., (s+1)!$, and $R^j(i) = \{\langle X, Y\rangle \mid J_{\langle 1,0\rangle}H_1(X, Y, i, j) \vee J_{\langle -1,0\rangle}H_1(X, Y, i, j) \vee J_{\langle \tau,0\rangle}H_1(X, Y, i, j)\}, j = 1, ..., (s+1)!$, $i = 0, 1, ..., s$.

The realization of JSM research is characterized by the functions $g_2(\langle v, y\rangle, \Omega(p), h)$ and $g_1(\langle z, y\rangle, \Omega(p), h)$, which are defined below.

The data structure of the ASSR JSM method used to determine facts and hypotheses about causes and predictions is based on two Boolean algebras $\mathcal{B}_1 = \langle 2^{U^{(1)}}, \varnothing, U^{(1)}, -, \cap, \cup\rangle$ и $\mathcal{B}_2 = \langle 2^{U^{(2)}}, \varnothing, U^{(2)}, -, \cap, \cup\rangle$ for representing objects (subobjects) and effects (sets of properties), respectively [4, 12].

We recall that in order to determine empirical regularities, the simplest case is considered such that there is the **only** cause V and effect Y such that the JSM reasoning is realized in two steps: for induction and for analogy (i.e., without iterations p.i.r.-1 and p.i.r.-2). Therefore, truth values $\langle v, 1\rangle$, $(\tau, 1)$ and $\langle v, 2\rangle$, $(\tau, 2)$ are generated, respectively, for p.i.r.-1 and p.i.r.-2[7].

To review the histories of possible worlds from $\overline{HPW}$, we parameterize the terms $\Delta(p), \tilde{\Delta}(p); \Omega(p), \tilde{\Omega}(p)$; introducing the variable h, we obtain $\Delta(p,h), \tilde{\Delta}(p,h)$; $\Omega(p,h), \tilde{\Omega}(p,h)$.

Let $\{\Omega\}$ denote the set $\{\Omega(0, h), \Omega(1, h), ..., \Omega(p, h), ..., \Omega(s, h)\}$ corresponding to the history of possible worlds h from $\overline{HPW}$. Then, we define the display $g_2 : (2^{U^{(1)}} \times 2^{U^{(2)}}) \times \{\Omega\} \times \overline{HPW} \to \{1, -1, 0, \tau\}$ as follows:

*Df.*4-2.

$$g_2(\langle V,Y\rangle, \Omega(p,h)) = \begin{cases} 1, \text{ if } J_{\langle 1,1\rangle}H_2(V,Y,p,h) \in \tilde{\Delta}^+(p,h) \\ -1, \text{ if } J_{\langle -1,1\rangle}H_2(V,Y,p,h) \in \tilde{\Delta}^-(p,h) \\ 0, \text{ if } J_{\langle 0,1\rangle}H_2(V,Y,p,h) \in \tilde{\Delta}^0(p,h) \\ \tau, \text{ if } J_{\langle \tau,1\rangle}H_2(V,Y,p,h) \in \tilde{\Delta}^\tau(p,h) \end{cases}.$$

Similarly, we define $g_1 : (2^{U^{(1)}} \times 2^{U^{(2)}}) \times \{\Omega\} \times \overline{HPW} \to \{1, -1, 0, \tau\}$:

$$g_1(\langle Z,Y\rangle, \Omega(p,h)) = \begin{cases} 1, \text{ if } J_{\langle 1,2\rangle}H_1(Z,Y,p,h) \in \tilde{\Omega}^+(p,h) \\ -1, \text{ if } J_{\langle -1,2\rangle}H_1(Z,Y,p,h) \in \tilde{\Omega}^-(p,h) \\ 0, \text{ if } J_{\langle 0,2\rangle}H_1(Z,Y,p,h) \in \tilde{\Omega}^0(p,h) \\ \tau, \text{ if } J_{\langle \tau,0\rangle}H_1(Z,Y,p,h) \in \tilde{\Omega}^\tau(p,h) \end{cases}.$$

[7] $(\tau, 1)$ and $(\tau, 2)$ are sets of truth values.

Table 1

$g_2(\langle V,Y\rangle, \Omega(p,h))$	$\Omega(0,h)\Omega(1,h)...\Omega(p,h)...\Omega(s,h)$
$\langle C_1^{/}, Q_1\rangle$	$\sigma_1(0) \quad \sigma_1(1)...\sigma_1(p)...\sigma_1(s)$
....	....
$\langle C_i^{/}, Q_i\rangle$	$\sigma_i(0) \quad \sigma_i(1)...\sigma_i(p)...\sigma_i(s)$
....	....
$\langle C_{r(h)}^{/}, Q_{r(h)}\rangle$	$\sigma_{r(h)}(0) \quad \sigma_{r(h)}(1)...\sigma_{r(h)}(p)...\sigma_{r(h)}(s)$

We note that $V_{in} = \{1, -1, 0\}$, $V_{ex} = \{t, f\}$ are the sets of actual ("internal") and logical ("external") truth values [4, 12]. The latter are used to determine J–functions [5]. We recall also that $(I)^\sigma_{x,y}(\Omega(p,h)) = \tilde{\Delta}^\sigma(p,h)$, and $\bar{O}_{x,y}(\Omega(p,h) = \tilde{\Omega}(p,h)$, where $\sigma = +, -, 0, \tau$; $\tilde{\Delta}^\sigma(p,h) \subseteq \tilde{\Delta}(p,h), \tilde{\Omega}^\sigma(p,h) \subseteq \tilde{\Omega}(p,h)$.

Let $\bar{Q}(h) = \{Q_1, ..., Q_{r(h)}\}, \bar{Q}(h) \subseteq 2^{U^{(2)}}$; then Table 1 sets $g_2(\langle V,Y\rangle, \Omega(p,h))$:

The application of the JSM reasoning to each $\Omega(i, h)$ generates $\tilde{\Delta}(i,h)$. Through $\tilde{\Delta}$, we denote the result of applying the JSM reasoning to the sequence FB(0, h), ..., FB(s, h) such that FB(0, h) $\subset ... \subset$ FB(s, h), which one-to-one corresponds to $\Omega(0, h)$, $\Omega(1, h)$, ..., $\Omega(s, h)$.

In this way,

$$\tilde{\Delta} = \bigcup_{i=0}^{s} \tilde{\Delta}(i,h), \text{where } \tilde{\Delta} \text{ one-to-one corresponds to} \left\{\langle C', Q_1\rangle, ..., \langle C_i^{/}, Q_1\rangle, ..., \langle C_{r(h)}^{/}, Q_{r(h)}\rangle\right\}.$$

As $\left|\Omega^\tau(0)\right| = m_0$, where $m_0 = l_0 + a + b + c$ [6], where a, b, c are three types of JSM reasoning errors and l_0 is the submission of **correct** (verified) predictions specifying $\Omega^\tau(0)$, which is one of the objectives of the JSM reasoning.

Three cases are possible: (1) $r(h) = m_0$, (2) $r(h) > m_0$, (3) $r(h) < m_0$. Almost the most possible case is (2).

Similarly to Table 1, we will consider Table 2 defining $g_1(\langle v, y\rangle, \Omega(p,h))$:

where $\left|\Omega^\tau(0)\right| = m_0$.

We note that $g_2(\langle V, Y\rangle, \Omega(p, h)) = v$, if and only if $J_{\langle v,1\rangle}H_2(V, Y, p, h) \in \tilde{\Delta}^\sigma(p,h)$, where $v = \begin{cases} 1, \text{ if } \sigma = + \\ -1, \text{ if } \sigma = -; \\ 0, \text{ if } \sigma = 0 \end{cases}$

$g_2(\langle V, Y\rangle, \Omega(p, h)) = \tau$, if and only if $J_{\langle \tau,1\rangle}H_2(V, Y, p, h) \in \tilde{\Delta}^\tau(p,h)$;

Table 2

$g_1(\langle z, y\rangle, \Omega(p, h)\rangle$	$\Omega(0, h)\Omega(1, h)\ldots\Omega(p, h)\ldots\Omega(s, h)$
$\langle C_1, Q_1\rangle$	$\theta_1(0)\quad \theta_1(1)\ldots\theta_1(p)\ldots\theta_1(s)$
....	....
$\langle C_i, Q_i\rangle$	$\theta_i(0)\quad \theta_i(1)\ldots\theta_i(p)\ldots\theta_i(s)$
....	....
$\langle C_{m_0}, Q_{m_0}\rangle$	$\theta_{m_0}(0)\quad \theta_{m_0}(1)\ldots\theta_{m_0}(p)\ldots\theta_{m_0}(s)$,

$g_1(\langle Z, Y\rangle, \Omega(p, h)) = v$, if and only if $J_{\langle v,2\rangle}H_2(Z, Y, p, h) \in \tilde{\Omega}^{\sigma}(p, h)$, where $\nu = \begin{cases} 1, \text{ if } \sigma = + \\ -1, \text{ if } \sigma = -, \\ 0, \text{ if } \sigma = 0 \end{cases}$

$g_1(\langle Z, Y\rangle, \Omega(p, h)) = \tau$, if and only if $J_{\langle \tau,2\rangle}H_1(Z, Y, p, h) \in \tilde{\Omega}^{\tau}(p, h)$.

Let us preliminarily and informally explain the meaning of the predicates of the preservation of the truth of hypotheses and the corresponding predicates of the preservation of the truth of predictions, using which the empirical laws contained in the histories of possible worlds from $\overline{HPW}$ will be defined.

Let $\langle C_i^{/}, Q_i\rangle$ for parameters p and h be performed by $J_{\langle 1,1\rangle}H_2(C_i^{/}, Q_i, p, h)$, and $Cd_1 = 1\ldots1$ sequence 1 is such that it corresponds to $\Omega(0, h)\Omega(1, h)\ldots\Omega(s, h)$ $\underbrace{1\quad 1\ldots1}_{s+1}$, and $\overline{Cd_2}$ is the set of sequences $1\ldots1$ such that they correspond to $\underset{1}{\Omega(0,h)}\underset{1}{\Omega(1,h)}\underset{1}{\Omega(s,h)}$ by virtue of feasibility of $J_{\langle 1,2\rangle}H_1(Z, Q_i, p, h)$ for all Z such that $C_i^{/} \subset Z$, then $Cd_1 \cdot \overline{Cd_2}$ will be called the set of codes of **empirical** law. Its elements will be $Cd = \underbrace{1\ldots1}_{s+1}\bullet\underbrace{1\ldots1}_{s+1}$, where "•" is the concatenation sign, and each $\sigma_i(p)$ corresponds to $\theta_i(p)$ such that $\sigma_i(p) = \theta_i(p)$, where $\sigma_i(p) = 1$. The code $Cd = \underbrace{-1\ldots-1}_{s+1}\cdot\underbrace{-1\ldots-1}_{s+1}$ is defined similarly. The codes $Cd = \sigma_i(0)\ldots\sigma_i(s)\bullet\ \theta_i(0)\ldots\theta_i(s)$, where $\sigma_i(p) = \theta_i(p)$, $\sigma_i(p) = 1(\sigma_i(p) = -1)$, will be called **purely regular**.

Thus, the pair $\langle C_i^{/}, Q_i\rangle$, performing $J_{\langle 1,1\rangle}H_2(V, Y, p, h)$, corresponds to the truth of $J_{\langle 1,2\rangle}H_1(Z, Q_i, p, h)$ for all Z such that $C_i^{/} \subset Z$.

The same holds for $\sigma_i(p) = -1$.

Let $\langle C_i^{/}, Q_i\rangle$ for parameters p and h such that $0 \le p \le q$ performs $J_{(\tau,1)}H_2(C_i^{/}, Q, p, h)$, and for parameters $q + 1 \le p \le s$ it performs $J_{\langle 1,1\rangle}H_2(C_i^{/}, Q, p, h)$; $Cd_1 = \underbrace{\tau\ldots\tau}_{q}\ \underbrace{1\ldots1}_{s+1-q}$ is a sequence $\tau\ldots\tau1\ldots1$ such that it corresponds to $\Omega(0, h)\Omega(1, h)\ldots\Omega(q, h)\Omega(q + 1, h)\ldots\Omega(s)$; and $\overline{Cd_2}$ is a set of sequences $\tau\ldots\tau\ 1\ldots1$ such that they correspond to $\underset{\tau}{\Omega(0,h)}\underset{\tau}{\Omega(1,h)}\underset{\ldots}{\ldots}\ \underset{\tau}{\Omega(q,h)}\underset{1}{\Omega(q+1,h)}\underset{\ldots}{\ldots}\ \underset{1}{\Omega(s)}$ by virtue of the feasibility of $J_{\langle 1,2\rangle}H_1(Z, Q_i, p, h)$ for all Z and p such that $q + 1 \le p \le s$, and $C_i^{/} \subset Z$, then $Cd_1 \cdot \overline{Cd_2}$ will be called the set of codes of **empirical** tendencies. Their elements will be $Cd = \underbrace{\tau\ldots\tau}_{q}\ \underbrace{1\ldots1}_{s+1-q}\bullet\underbrace{\tau\ldots\tau}_{q}\ \underbrace{1\ldots1}_{s+1-q}$, where "•" is the concatenation sign, and each $\sigma_i(p)$, where $0 \le p \le q$, corresponds to $\theta_i(p)$ such that $\sigma_i(p) = \theta_i(p)$; and each $\sigma_i(p)$, where $q + 1 \le p \le s$, corresponds to θ_i such that $\sigma_i(p) = \theta_i(p)$.

$Cd = \tau\ldots\tau - 1\ldots-1 \bullet \tau\ldots\tau - 1\ldots-1$ is defined similarly for $J_{(\tau,1)}H_2(C_i^{/}, Q_i, p, h)$, $J_{(\tau,2)}H_1(Z, Q_i, p, h)$ for $0 \le p \le q$, respectively, and all Z; as well as $J_{\langle -1,1\rangle}H_2(C_2^{/}, Q_i, p, h)$, $J_{\langle -1,2\rangle}H_1(Z, Q_i, p, h)$ for $q + 1 \le p \le s$, respectively, and all Z.

We define weak empirical tendencies for the condition $q \ge s + 1 - q$.

To formalize empirical regularities [1, 2] (empirical laws, tendencies, and weak tendencies), we introduce the definitions of the corresponding predicates expressing the preservation of types of truth values (1, −1, τ) for hypotheses about the causes and their corresponding hypotheses about predictions for extensions of the FB(p).

Df. 5-2. For a fixed strategy of JSM reasoning $Str_{x,y}$, the only cause V for the cases where there are no iterations of plausible inference rules (p.i.r.-1 and p.i.r.-2), we formulate the definition of a predicate that preserves the type of truth value "1" for FB(0) ⊂ ... ⊂ FB(s): $L_2^{+}(V, Y, p, s, h) \rightleftharpoons (((0 \le p \le s)\ \&\ (\rho^{+}(s) \ge \bar{\rho}^{+})) \to (J_{\langle 1,1\rangle}H_2(V, Y, p, h) \in \tilde{\Delta}^{+}(p, h))\ \&\ (J_{\langle 1,1\rangle}H_2(V, Y, 0, h) \in \tilde{\Delta}^{+}(0)))$, where $(I)_{x,y}^{+}(\Omega(p, h)) = \tilde{\Delta}^{+}(p, h)$; $\hat{L}_2^{+}(V, Y, p, s, h) \rightleftharpoons L_2^{+}(V, Y, p, s, h)\ \&\ (\rho^{+}(0) \le \ldots \le \rho^{+}(s))$.

We define $L_2^{-}(V, Y, p, s, h)$ and $\hat{L}_2^{-}(V, Y, p, s, h)$ similarly.

Similarly to *Df.*5-2, we formulate the definition of a predicate of conservation of the type of truth value for hypotheses about prediction with respect to FB(0) ⊂ ... ⊂ FB(s):

***Df.*6-2.** $L_1^{+}(Z, Y, p, s, h) \rightleftharpoons ((((0 \le p \le s)\ \&\ (\rho^{+}(s) \ge \bar{\rho}^{+})) \to (J_{\langle 1,2\rangle}H_1(Z, Y, p, h) \in \tilde{\Omega}^{+}(p, h))\ \&\ (J_{\langle 1,2\rangle}H_1(Z, Y, 0, h) \in \tilde{\Omega}^{+}(0)))$, where $\bar{O}_{x,y}(\Omega(p, h)) = \tilde{\Omega}(p, h), \tilde{\Omega}^{\sigma}(p, h) \subseteq \tilde{\Omega}(p, h), \sigma = +, -, 0, \tau$.

$\hat{L}_1^+(Z,Y,p,s,h) \rightleftharpoons L_1^+(Z,Y,p,s,h)$ & $(\rho^+(0) \le \ldots \le \rho^+(s))$.

We define $L_1^-(Z,Y,p,h)$ and $\hat{L}_1^-(Z,Y,p,s,h)$ similarly.

We note that $L_2^\sigma(V,Y,p,h)$ and $L_1^\sigma(Z,Y,p,s,h)$, where $\sigma = +, -$, correspond to $Cd = Cd_1 \cdot \overline{Cd}_2$, discussed above, where $Cd_1 = \nu\ldots\nu$, and $\nu = 1, -1$.

*Df.*5-2 and *Df.*6-2 will be used to determine causal forcings of the investigated effect, as well as to determine empirical laws.

*Df.*7-2 and *Df.*8-2, as formulated below will be used to determine empirical tendencies such that they correspond to $Cd = Cd_1 \cdot \overline{Cd}_2$, where $Cd_1 = \tau\ldots\tau\nu\ldots\nu$, and $\nu = 1, -1$, $\overline{Cd}_2$ is a set of similar codes for all Z such that $C_i^/ \subset Z$, where $C_i^/$ represents the alleged cause of the effect being the value Y in $g_2(\langle V, Y\rangle, \Omega(p, h)$ and $g_1(\langle Z, Y\rangle, \Omega(p,h))$.

***Df.*7-2.** $L_{2,\tau}^+(V,Y,p,s,h) \rightleftharpoons \exists q((((\rho^+(s) \ge \overline{\rho}^+)$ & $((J_{\langle 1,1\rangle}H_2(V,Y,p,s,h) \in \tilde{\Delta}^+(p,h)) \vee (J_{(\tau,1)}H_2(V,Y,p,h) \in \tilde{\Delta}^\tau(p,h)))$ & $(0 \le p \le s)) \to ((((0 \le p \le q)$ & $(2(q+1) < s)) \to g_2(\langle V, Y\rangle, \Omega(p,h)) = \tau)$ & $((q + 1 \le p \le s) \to g_2(\langle V, Y\rangle, \Omega(p, h)) = 1)))$ & $((J_{(\tau,1)}H_2(V,Y,0,h) \in \tilde{\Delta}^\tau(0,h))$ & $(J_{\langle 1,1\rangle}H_2(V,Y,q+1,h) \in \tilde{\Delta}^+(q+1)))$, $\hat{L}_{2,\tau}^+(V,Y,p,s,h) \rightleftharpoons L_{2,\tau}^+(V,Y,s,p,h)$ & $(\rho^+(0) \le \ldots \le \rho^+(s))$.

Similarly, we define $L_{2,\tau}^-(V,Y,p,s,h)$ and $\hat{L}_{2,\tau}^-(V,Y,p,s,h)$.

***Df.*8-2.** $L_{1,\tau}^+(Z,Y,p,s,h) \rightleftharpoons \exists q((((\rho^+(s) \ge \overline{\rho}^+)$ & $((J_{\langle 1,2\rangle}H_1(Z,Y,p,h) \in \tilde{\Omega}^+(p,h)) \vee (J_{\langle \tau,2\rangle}H_1(Z,Y,p,h) \in \tilde{\Omega}^\tau(p,h))))$ & $(0 \le p \le s)) \to ((((0 \le p \le q)$ & $(2(q+1) < s)) \to g_1(\langle Z, Y\rangle, \Omega(p, h)) = \tau)$ & $((q + 1 \le p \le s) \to g_1(\langle Z, Y\rangle, \Omega(p,h)) = 1))$ & $((J_{\langle \tau,2\rangle}H_1(Z,Y,0,h) \in \tilde{\Omega}^\tau(0))$ & $(J_{\langle 1,2\rangle}H_1(Z,Y,q+1,h) \in \tilde{\Omega}^+(q+1,h)))$, $\hat{L}_{1,\tau}^+(Z,Y, p, s, h) \rightleftharpoons L_{1,\tau}^+(Z,Y,s,p,h)$ & $(\rho^+(0) \le \ldots \le \rho^+(s))$.

Similarly, we define $L_{1,\tau}^-(Z,Y,p,s,h)$ and $\hat{L}_{1,\tau}^-(Z,Y,s,p,h)$.

Replacing the condition $2(q + 1) < s$ to $2(q + 1) \ge s$ in $L_{2,\tau}^\sigma(V,Y,p,h)$ and $L_{1,\tau}^\sigma(Z,Y,p,h)$, where $\sigma = +, -$, we obtain the definitions $\overline{L}_{2,\tau}^\sigma(V,Y,p,h)$ and $\overline{L}_1^\sigma(Z,Y,p,h)$, which express weak empirical tendencies.

Using the MJL language, we define the estimation function $V[\varphi]$ of the formulas of the JL language.

Remark 6-2. In [4], a distinction was made between correspondent and coherent truth values. Corresponding truth values [25, 26] are determined by the conditions of T.A. Tarski: “the consistency of the statement with the state of affairs.” The coherent truth values of the statement φ are determined by the feasibility of some consistent conditions that characterize φ.

Parcels of p.i.r.-1 (induction) and p.i.r.-2 (analogy) are the basis for the derivability of the effects and assignment of truth values $\langle\nu, 1\rangle$, sets of truth values $(\tau, 1)$ for p.i.r.-1 and $\langle\nu, 2\rangle$ and $(\tau, 2)$ for p.i.r.-2, where $\nu = 1, -1, 0$, to them.

Parcels of p.i.r.-1 premises carry out forcing hypotheses about the causes: **forcing based on fact similarity** (FS). As for parcels of p.i.r.-2, they carry out **local causal forcing** of predictive hypotheses (LCF) through hypotheses of causes.

FS and LCF are conditions for generating **coherent** truth values.

Assessments $H_1(C, Q, p, h)\,\langle\nu, 0\rangle$ and $(\tau, 0)$, where $\nu = 1, -1$ are types of **correspondent** truth values for $R^h(p)$, where $p = 0, 1, \ldots, s$; $h = 1, 2, \ldots, (s + 1)!$, and C, Q are constants. By virtue of this, we put $\langle 1, 0\rangle = t$, $\langle -1, 0\rangle = f$, where t,f are truth values of two-valued logic, and $(\tau, 0) = \tau$.

We will determine $V[\varphi]$ for a fixed $Str_{x,y}$.

1^0. $V[H_1(C, Q, p, h)] = \langle v, 0\rangle$ for $R^h(p)$, where $v = 1, -1$; C, Q are constants, and $p = 0, 1, \ldots, s$; $h = 1, \ldots, (s + 1)!$

2^0. $V[H_2(C', Q, p, h)] = \langle 1, 1\rangle$, if and only if $J_{(\tau,0)}H_2(C',Q,p,h)$ & $M_{x,0}^+(C',Q)$ & $\neg M_{y,0}^-(C',Q)$;

3^0. $V[H_2(C', Q, p, h)] = \langle -1, 1\rangle$, if and only if $J_{(\tau,0)}H_2(C',Q,p,h)$ & $\neg M_{x,0}^+(C',Q)$ & $M_{y,0}^-(C',Q)$;

4^0. $V[H_2(C', Q, p, h)] = \langle 0, 1\rangle$, if and only if $J_{(\tau,0)}H_2(C',Q,p)$ & $M_{x,0}^+(C',Q)$ & $M_{y,0}^-(C',Q)$;

5^0. $V[H_2(C', Q, p, h)] = (\tau, 1)$, if and only if $J_{(\tau,0)}H_2(C',Q,p,h)$ & $\neg M_{x,0}^+(C',Q)$ & $\neg M_{y,0}^-(C',Q)$;

6^0. $V[H_1(C, Q, p, h)] = \langle 1, 2\rangle$, if and only if $J_{(\tau,1)}H_1(C,Q,p,h)$ & $\mathrm{P}^+(C,Q)$;

7^0. $V[H_1(C, Q, p, h)] = \langle -1, 2\rangle$, if and only if $J_{(\tau,1)}H_1(C,Q,p,h)$ & $\mathrm{P}^-(C,Q)$;

8^0. $V[H_1(C, Q, p, h)] = \langle 0, 2\rangle$, if and only if $J_{(\tau,1)}H_1(C,Q,p,h)$ & $\mathrm{P}^\circ(C,Q)$;

9^0. $V[H_1(C, Q, p, h)] = (\tau, 2)$, if and only if $J_{(\tau,1)}H_1(C,Q,p,h)$ & $\mathrm{P}^\tau(C,Q)$;

10^0. JL formulas formed by $-$, $\cap$, $\cup$, $\subseteq$, $=$ are assessed in the standard manner;

11^0. $J_{\bar{v}}\varphi = \begin{cases} t, \text{ if } V[\varphi] = \bar{v} \\ f, \text{ otherwise} \end{cases}$, where φ is the JL formula.

12^0. if φ, ψ are JL formulas such that the predicates H_1 and H_2 are within the domain of J-operators or do not contain these predicates, then the formulas $\neg\varphi, (\varphi \,\&\, \psi), (\varphi \vee \psi), (\varphi \to \psi)$ are assessed in the standard manner, as two-valued logic formulas.

13^0. if $\varphi(X)$ is a JL formula such that X enters freely and X is within the domain of the J-operator or does not contain predicates H_1 and H_2, then $V[\forall X\varphi(X)] = t$, if and only if $V[\varphi(C)] = t$ for every $C \in 2^{U^{(1)}}$;

14^0. if $\varphi(Y)$ is a JL formula such that Y enters φ freely and Y is within the domain of the J-operator or does not contain predicates H_1 and H_2, then $V[\forall Y\varphi(Y)] = t$, if and only if $V[\varphi(Q)] = t$ for each $Q \in 2^{U^{(2)}}$.

Similarly, we define the assessment $V[\exists X\varphi(X)]$ and $V[\exists Y\varphi(Y)]$ in 15^0, 16^0;

17^0. $V[\forall p\varphi(p)] = t$, if and only if $V[\varphi(\bar{p})] = t$ for all values $\bar{p}$ of the variable p such that $0 \le \bar{p} \le \bar{s}$, relevant $R^h(\bar{p})$, where h is the history number of possible worlds from $\overline{HPW}$;

18^0. $V[\exists p\varphi(p)] = t$, if and only if exists $\bar{p}$ such that $0 \le \bar{p} \le \bar{s}$ and $V[\varphi(\bar{p})] = t$ relative to $R^h(\bar{p})$;

19^0. $V[\forall h\varphi(h)] = t$, if and only $V[\varphi(\bar{h})] = t$ for all $R^h(p)$, where $1 \le \bar{h} \le (\bar{s} + 1)$ and $0 \le \bar{p} \le \bar{s}$, where $\bar{p}$ is the value of p;

20^0. $V[\exists h\varphi(h)] = t$, if and only if there exists $\bar{h}$ such that $V[\varphi(\bar{h})] = t$ for $R^{\bar{h}}(p)$ and all $\bar{p}$ $0 \le \bar{p} \le \bar{s}$;

21^0. $V[\forall n\varphi(n)] = t$, where n is variables of grade 3 if and only if $V[\varphi(\bar{n})] = t$ for all $\bar{n} = 0, 1, \ldots$;

22^0. $V[\exists n\varphi(n)] = t$, if and only if there is $\bar{n}$ such that $V[\varphi(\bar{n})] = t$.

Using predicate pairs $L_2^\sigma(V,Y,p,h), L_1^\sigma(Z,Y,p,h)$; $\hat{L}_2^\sigma(V,Y,p,h), \hat{L}_1^\sigma(Z,Y,p,h)$; $L_{2,\tau}^\sigma(V,Y,p,h)$, $L_{1,\tau}^\sigma(Z,Y,p,h)$; $\hat{L}_{2,\tau}^\sigma(V,Y,p,h), \hat{L}_{1,\tau}^\sigma(Z,Y,p,h)$; $\overline{L}_{2,\tau}^\sigma(V,Y,p,h), \overline{L}_{1,\tau}^\sigma(Z,Y,p,h)$; $\hat{\overline{L}}_{2,\tau}^\sigma(V,Y,p,h), \hat{\overline{L}}_{1,\tau}^\sigma(V,Y,p,h)$, we define **prolonged** causal forcings (PCF) for $\sigma = +, -$, which are the applied local causal forcings for the histories of possible worlds HPW $R^h(0)$, $R^h(1)$, ..., $R^h(s)$.

PCF expresses the condition for **preserving** local causal forcing for all possible worlds $R^h(p)$, beginning with $R^h(0)$ for the relevant HPW_h. The dependencies of predicate pairs L_2^σ, L_1^σ and $\hat{L}_2^\sigma, \hat{L}_1^\sigma$; $L_{2,\tau}^\sigma, L_{1,\tau}^\sigma$ and $\hat{L}_{2,\tau}^\sigma, \hat{L}_{1,\tau}^\sigma$; $\overline{L}_{2,\tau}^\sigma, \overline{L}_{1,\tau}^\sigma$ and $\hat{\overline{L}}_{2,\tau}^\sigma, \hat{\overline{L}}_{1,\tau}^\sigma$ express empirical regularities (ERs), empirical tendencies (ETs) and weak empirical tendencies (WETs), respectively.

***Df.*9-2.** The PCF conditions for **empirical regularities** are

(1) $\exists h\exists s\forall V\forall Y\forall Z\forall p((L_2^\sigma(V,Y,p,s,h) \,\&\, (V \subset Z) \,\&\, P(Z,p,h)) \to L_1^\sigma(Z,Y,p,s,h))$, where $P(Z, p, h) \rightleftharpoons \neg\exists V_0((J_{\langle -1,1\rangle}H_2(V_0, Z, p, h) \vee J_{\langle 0,1\rangle}H_2(V_0, Z, p, h)) \,\&\, (V_0 \subset Z))$;

(2) $\exists h\exists s\forall V\forall Y\forall Z\forall p((\hat{L}_2^\sigma(V,Y,p,s,h) \,\&\, (V \subset Z) \,\&\, P(Z,p,h) \to \hat{L}_1^\sigma(Z,Y,p,s,h))$, where $\sigma = +, -$.

***Df.*10-2.** The PCF conditions for **empirical tendencies** are

(3) $\exists h\exists s\forall V\forall Y\forall Z\forall p((L_{2,\tau}^\sigma(V,Y,p,s,h) \,\&\, (V \subset Z) \,\&\, (V \subset Z) \,\&\, P(Z,p,h)) \to L_{1,\tau}^\sigma(Z,Y,p,s,h))$;

(4) $\exists h\exists s\forall V\forall Y\forall Z\forall p((\hat{L}_{2,\tau}^\sigma(V,Y,p,s,h) \,\&\, (V \subset Z) \,\&\, P(Z,p,h)) \to \hat{L}_{1,\tau}^\sigma(Z,Y,p,s,h)$.

***Df.*11-2.** The PCF conditions for **weak empirical tendencies** are

(5) $\exists h\exists s\forall V\forall Y\forall Z\forall p((\overline{L}_{2,\tau}^\sigma(V,Y,p,s,h) \& (V \subset Z) \,\&\, P(Z,p,h)) \to \overline{L}_{1,\tau}^\sigma(Z,Y,p,s,h))$,

(6) $\exists h\exists s\forall V\forall Y\forall Z\forall p((\hat{\overline{L}}_{2,\tau}^\sigma(V,Y,p,s,h) \,\&\, (V \subset Z) \,\&\, P(Z,p,h)) \to \hat{\overline{L}}_{1,\tau}^\sigma(Z,Y,p,s,h))$.

The following Proposal is an MJL theorem formulated regarding the histories of possible worlds. $R^h(0)$, $R^h(1)$,..., $R^h(s)$.

Proposal 1-2. Conditions for prolonged causal forcings PCF determined by predicate pairs

$$L_2^\sigma(V,Y,p,s,h), L_1^\sigma(Z,Y,p,s,h);$$
$$\hat{L}_2(V,Y,p,s,h), \hat{L}_1^\sigma(Z,Y,p,s,h);$$
$$L_{2,\tau}^\sigma(V,Y,p,s,h), L_{1,\tau}^\sigma(Z,Y,p,s,h);$$
$$\hat{L}_{2,\tau}^\sigma(V,Y,p,s,h), \hat{L}_{1,\tau}^\sigma(Z,Y,p,s,h);$$
$$\overline{L}_{2,\tau}^\sigma(V,Y,p,s,h), \overline{L}_{1,\tau}^\sigma(Z,Y,p,s,h);$$
$$\hat{\overline{L}}_{2,\tau}^\sigma(V,Y,p,s,h), \hat{\overline{L}}_{1,\tau}^\sigma(Z,Y,p,s,h)$$

are true about the histories of possible worlds HPW $R^h(0)$, $R^h(1)$, ..., $R^h(s)$ for fixed strategies of JSM reasoning.

To prove Proposal 1-2 with respect to HPW, we reformulate p.i.r.-1 and p.i.r.-2 for predicates $H_2(V, Y, p, h)$ and $H_1(Z, Y, p, h)$. Without loss of generality, we consider the case $\sigma = +, \bullet$.

We obtain $P^+(Z, Y) \rightleftharpoons \exists V(J_{\langle 1,1\rangle}H_2(V, Y, p, h) \,\&\, (V \subset Z) \,\&\, \neg\exists V_0((J_{\langle -1,1\rangle}H_2(V_0, Y, p, h) \vee J_{\langle 0,1\rangle}H_2(V_0, Y, p, h)) \,\&\, (V \subset Z))$.

Here, we consider the case of JSM reasoning with the existence of a single cause and the absence of iterations of the plausible inference rules, which is limited to the application of truth values $\langle v, n\rangle$, where $v = 1, -1, 0$, their set (τ, n), where $n = 0, 1, 2$.

We reformulate $(II)_{x,y}^{+}$ and $(I)_{x,y}^{+}$, by introducing the parameters p and h:

$$(II)^{+}_{x,y}\ \frac{J_{(\tau,1)}H_1(Z,Y,p,h)\ \&\ \mathrm{P}^{+}(Z,Y,p,h)}{J_{\langle 1,2\rangle}H_1(Z,Y,p,h)},$$

$$(I)^{+}_{x,y}\ \frac{J_{(\tau,0)}H_2(V,Y,p,h)\ \&\ M^{+}_{x,0}(V,Y,p,h)\ \&\ \neg M^{-}_{y,0}(V,Y,p,h)}{J_{\langle 1,1\rangle}H_2(V,Y,p,h)}.$$

$(II)^{\sigma}_{x,y}$ and $(I)^{\sigma}_{x,y}$ for $\sigma = -, 0, \tau$ are formulated similarly. We note that for p.i.r.-1 and p.i.r.-2, the parcels and their consequences are reversible [28].

We consider a fixed but arbitrary strategy of JSM reasoning $Str_{x,y}$.

We consider PCF for empirical regularities (1) from *Df.*9-2 and (2) from *Df.*10-2.

We suppose that the antecedent (1) is true and there are $\overline{h}$ and $\overline{s}$ – values h and s. Let C', Q, C and $\overline{p}$ be any values V, Y, Z and p, respectively. Let $L_2^{+}(C',Q,\overline{p},\overline{s},\overline{h})\ \&\ (C' \subset C)\ \&\ \mathrm{P}(C,\overline{p},\overline{h})$, be true, then $J_{\langle 1,1\rangle}H_2(C',Q,\overline{p},\overline{h})$ is true by virtue of the definition $L_2^{+}(V,Z,p,s,h)$.

Next, the truth of $\mathrm{P}(C,\overline{p},\overline{h})$ entails truth $\neg\exists V_0((J_{\langle -1,1\rangle}H_2(V_0, Y, p, h) \vee J_{\langle 0,1\rangle}H_2(V_0, Y, p, h))\ \&\ (V_0 \subset C))$ entails truth $\mathrm{P}^{+}(Z, Y, p, h)$, and therefore $L_1^{+}(C,C',\overline{p},\overline{h})$ is true.

Thus, if $V[L_2^{+}(C',Q,\overline{p},\overline{s},\overline{h})\ \&\ (C' \subset C)$ & $V[L_2^{+}(C',Q,\overline{p},\overline{s},\overline{h})\ \&\ (C' \subset C)\ \&\ \mathrm{P}(C,\overline{p},\overline{h})] = t$, then $V[L_1^{+}(C,Q,\overline{p},\overline{s},\overline{h})] = t$, Q.E.D.

Proposal (2) is proved similarly, as well as the case with $\sigma = -$ for (1) and (2).

We consider the conditions CF for (1) and (2) for empirical tendencies.

We suppose that antecedent (3) is true for arbitrary constants $\overline{h},\overline{s},C',Q,C,\overline{p}$ $V[L_{2,\tau}^{+}(C',Q,\overline{p},\overline{s})\ \&\ (C' \subset C)\ \&\ \mathrm{P}(C,\overline{p},\overline{h})] = t$. We consider two cases when $0 \le p \le \overline{q}$ and $\overline{q}+1 \le p \le \overline{s}$. If $0 \le p \le \overline{q}$, then $V[J_{(\tau,1)}H_2(C',Q,\overline{p},\overline{h})] = t$, then by virtue of uniqueness of C' (by assumption) and the definition of the predicate $\mathrm{P}^{+}(Z, Y, p, h)$ and $V[P(C,p,\overline{s})] = t$ we obtain $V[J_{\langle -1,1\rangle}H_2(C',Q,p,\overline{h})] = f$ and $V[J_{\langle 0,1\rangle}H_2(C',Q,p,\overline{h})] = f$, $V[J_{(\tau,1)}H_2(C',Q,p,\overline{h})] = t$ and, hence, $V[J_{(\tau,1)}H_1(C,Q,p,\overline{h})] = t$ by virtue of the Fifth exceptional law of four-digit logic [29].

If $q+1 \le p \le \overline{s}$, then $V[J_{\langle 1,1\rangle}H_2(C',Q,p,\overline{h})] = t$ $V[J_{\langle \nu,1\rangle}H_2(C',Q,p,\overline{h})] = f$, where $\nu = -1$, 0, and $V[J_{\langle \tau,1\rangle}H_2(C',Q,p,\overline{h})] = f$; therefore, by definition $\mathrm{P}^{+}(Z, Y, p, h)$ we obtain that $V[J_{\langle 1,1\rangle}H_2(C',Q,p,\overline{h})] = t$.

The proofs for the case $\sigma = -$ and conditions (4), (5), (6) are similar.

We consider the conditions of causal forcings (CF) (1)–(6), denoting them by A_j^{σ}, $1 \le j \le \sigma$, where $\sigma = +, -$. Replacing in A_j^{σ} variables h, s, V, Y by their values $\overline{h},\overline{s},C',Q$, accordingly, we obtain $A_j^{\sigma}(\overline{h},\overline{s},C',Q)$, that is, their realizations $\mathrm{RCF_j}$.

$A_j^{\sigma}(\overline{h},\overline{s},C',Q)$ will be called **empirical pre-regularities** (PERs): empirical pre-laws (PELs), pre-tendencies (PETs), and weak pre-tendencies (PWETs).

Without loss of generality, we consider CF (1) and introduce the corresponding definitions, which we then extend to the case of CF (2)–(6).

$D_{2,1}^{+}(V,Y,Z,p,s,h) \rightleftharpoons L_2^{+}(V,Y,p,s,h)\ \&\ (V \subset Z)\ \&\ P(Z,p,h)$, where $P(Z, p, h) \rightleftharpoons \neg\exists V_0(J_{\langle -1,1\rangle}H_2(V_0, Y, p, s, h) \vee J_{\langle 0,1\rangle}H_2(V_0, Y, p, h))\ \&\ (V_0 \subset Z))$.

Realization (1) $\mathrm{RCF_1}$:

$\forall Z\forall p((L_2^{+}(C',Q,p,\overline{s},\overline{h})\ \&\ (C' \subset Z)\ \&\ P(Z,p,\overline{h})) \to L_1^{+}(Z,Q,p,\overline{h}))$. Through $D_1^{+}(Z,Y,p,h)$ we denote $L_1^{+}(Z,Y,p,h)$: $D_{1,1}^{+}(Z,Y,p,h) \rightleftharpoons L_1^{+}(Z,Y,p,h)$. Then, we obtain $\exists h\exists s\forall V\forall Y\forall Z\forall p((D_{2,1}^{+}(V,Y,Z,p,h) \to D_{1,1}^{+}(Z,Y,p,h))$, $D_{2,1}^{+} = \{\langle Z,p\rangle \mid D_2^{+}(C',Q,Z,p,\overline{s},\overline{h})\}$, $D_{1,1}^{+} = \{\langle Z,p\rangle \mid L_1^{+}(Z,Q,p,\overline{s},\overline{h})\}$.

We note that $D_{2,1}^{+}$ and $D_{1,1}^{+}$ are binary relations expressed in MJL, which we expand accordingly.

We note that by virtue of CF (1): $D_{2,1}^{+} \subseteq D_{1,1}^{+}$, which is a consequence of Proposal 1-2.

We accept the following *Assumption* (*) : $\neg(D_{2,1}^{+} = \Lambda)$, where Λ is the empty relation.

We obtain the result of *Assumption* (*) and Proposals 1-2: $\neg(D_{1,1}^{+} = \Lambda)$.

Obviously, similar statements hold for $D_{2,j}^{\sigma}, D_{1,j}^{\sigma}$, where $\sigma = +, -$, and $j = 1, \ldots, 6$: $\neg(D_{2,j}^{\sigma} = \Lambda)$ are assumption, then $\neg(D_{1,j}^{\sigma} = \Lambda)$, as $D_{2,j}^{\sigma} \subseteq D_{1,j}^{\sigma}$.

We recall now the initial conditions for applying the ASSR JSM method:

1^0. $\Omega(0, 1), \Omega(1, 1), ..., \Omega(p, 1), ..., \Omega(s, 1)$, that is, the "history of the real world" $\Omega(0, 1)$, where $\Omega(0, 1) \subset \Omega(1, 1) \subset ... \subset \Omega(p, 1) \subset ... \subset \Omega(s, 1)$;

2^0. $\Omega^\tau(0, 1)$, where $\Omega^\tau(0, 1) = \Omega^\tau(p, h)$ for all $0 \leq p \leq s$, $1 \leq h \leq (s + 1)!$;

3^0. $\overline{HPW}$, where $|\overline{HPW}| = (s + 1)!$. $\overline{HPW}$ is a finite set of histories of finite possible worlds (PW);

4^0. $\overline{Str}$ is a set of strategies of JSM reasoning [6, 13].

We now state the **applicability** condition of the ASSR JSM method.

***Df.*12-2.** We say that the ASSR JSM method is **applicable** if there are strategies Str_{x_1,y_1} and $D^+_{2,j}$ or Str_{x_2,y_2} and $D^-_{2,j}$ such that $\neg(D^+_{2,j} = \Lambda) \vee \neg(D^-_{2,j} = \Lambda)$, where $1 \leq i, j = 6$.

*Df.*12-2 expresses the fact that empirical pre-regularities (PERs) are detectable, which means the truth of the applicability conditions of the ASSR JSM method.

Remark 7-2. Determination of the applicability of the ASSR JSM method can be enhanced by the addition of a **consistency** condition for realizations of $A^\sigma_j(\overline{h}, \overline{s}, C', Q)$ and the set of hypotheses generated for all PW histories from $\overline{HPW}$ [1].

Remark 8-2. CF realizations will be called **empirical prenomological statements**. This term is some modification of the term "nomological statement" introduced by *H.* Reichenbach in [30, 31]. Empirical prenomological statements are defined for some HPW from $\overline{HPW}$, whereas, below we define **empirical nomological statements** such that they are true for all HPW from $\overline{HPW}$.

The empirical nature of prenomological statements is characterized by the condition of **non-emptiness** of the antecedent $\neg(D^\sigma_{2,j} = \Lambda)$, as well as the use of constants $\in$ and $L^\sigma_1(Z, Q, p, \overline{s}, \overline{h})$, that is, the values of variables h and s, respectively. Examples of prolonged RCF are $\forall Z \forall p((L^\sigma_2(C', Q, p, \overline{s}, \overline{h}) \& (C \subset Z) \& P(Z, p, \overline{h})) \rightarrow L^\sigma_1(Z, Q, p, \overline{h}))$, where $\sigma = +, -$, and $\neg(D^\sigma_{2,1} = \Lambda)$ is true.

In §2, we considered prolonged causal forcings (PCF) [1, 2] and their corresponding realizations (RCF), expressed by empirical prenomological statements that are defined for fixed histories of possible worlds HPW from a given $\overline{HPW}$. Below, in §3 we define the **integral causal forcings** (ICF) for all HPWs that form $\overline{HPW}$, where $|\overline{HPW}| = (s + 1)!$.

3. INTEGRAL CAUSAL FORCINGS AND A SET OF EMPIRICAL REGULATIONS *ER*

Empirical pre-regularities (PERs) represented by empirical pre-nomological statements are defined for the histories of possible worlds HPW, which are the values of the variable *h*. The empirical regularities (ERs) discussed in this section are defined for all histories of possible worlds from the set $\overline{HPW}$ through realizations of integral causal forcings (ICF), which are representable by **empirical nomological statements**.

The ICF of the studied effect is determined by the set CF determined for all HPW from $\overline{HPW}$ through **basic** CF A^σ_j, where $1 \leq j \leq 6$, and $\sigma = +, -$, considered in §2.

The set of all ICFs denoted by $\overline{ICF}$, forms the **intensional** of the concept of the empirical regularity IntER, and the corresponding set of realizations $\overline{ICF}$ is an extensional ExtER corresponding to IntER and formed by a set of strategies $\overline{Str}$ of JSM reasoning and their applications to all histories of possible worlds from $\overline{HPW}$.

Using the predicate pairs $L^\sigma_2, L^\sigma_1; \hat{L}^\sigma_2, \hat{L}^\sigma_1; L^\sigma_{2,\tau}, L^\sigma_{1,\tau}$; $L^\sigma_2, L^\sigma_1; \hat{L}^\sigma_2, \hat{L}^\sigma_1; L^\sigma_{2,\tau}, L^\sigma_{1,\tau}$; basic CF$A^\sigma_j$, were defined, where $1 \leq j \leq \sigma$, $\sigma = +, -$. CFA^σ_j correspond to their RCF realization for pairs $\langle C', Q \rangle$, where *C'* is the carrier of the cause, and *Q* is the carrier of the effect.

We note that each HPW from $\overline{HPW}$, where $|\overline{HPW}| = (s + 1)!$, corresponds to some A^σ_j and its realization, which corresponds to some code of empirical regularity $Cd = Cd_1 \cdot \overline{Cd_2}$, where $\overline{Cd_2}$ is the set of codes for all *Z* such that $V \subset Z$ (*V* is the variable for the carrier of the cause).

The following conditions are the basis for ordering of A^σ_j:

(1) the type *Cd* is a nonempty regular code ($v = 1, -1$): $\langle v, v \rangle$ *, $\langle \tau, v \rangle$ *, which is the code of the **initial** HPW;

(2) types of nonempty regular codes that are **descendants (heirs)** of the initial Cd, which are $\langle v, v \rangle \mid \langle \tau, v \rangle, \langle \tau, v \rangle$ such that

(3) $\langle \tau, v \rangle$ has $2q < s + 1$ repetitions of τ, $\langle \tau, v \rangle$ has $2q \geq s + 1$ repetitions of τ, which characterizes empirical tendencies (ET) and weak empirical tendencies (WET), respectively;

(4) there is one of the possibilities, that is, the satisfiability of the monotony condition

(5) $\rho^\sigma(p)$ or its unsatisfiability, denoted by *M* and $\neg M$, respectively.

We note also that $\langle v, v \rangle$ and $\langle \tau, v \rangle$ denote $v...v \bullet v...v$ and $\tau...\tau v...v \bullet \tau...\tau v...v$, respectively, with a "length" $2(s + 1)$; $\langle v, v \rangle$* and $\langle \tau, v \rangle$* denote the selected *Cd* of the initial empirical regularities.

We recall that $Cd = Cd_1 \bullet \overline{CD}_2$, where $|Cd_1| = |Cd_2| = s$.

Table 3 presents 14 possible integral causal forcings – A_a^σ, A_b^σ, A_c^σ, A_d^σ, A_e^σ, A_f^σ, A_g^σ, A_h^σ, A_i^σ, A_j^σ, A_k^σ, A_l^σ, A_m^σ, A_n^σ, where $\sigma = +, -$. These ICFs are characterized by PCF generators (prolonged CF), the initial PCFs (their *Cd* codes), *Cd* descendants of the initial PCFs, and the monotony of the degree of abduction of the first kind, that is, functions $\rho^\sigma(p)$.

ICF implementations are **empirical regularities**: regularities of preserving determinations by hypotheses about the causes of hypotheses about predictions, which are expressed by predicates $L_2^\sigma, L_1^\sigma; \hat{L}_2^\sigma, \hat{L}_1^\sigma; L_{2,\tau}^\sigma, L_{1,\tau}^\sigma; \overline{L}_{2,\tau}^\sigma, \overline{L}_{1,\tau}^\sigma, \overline{\hat{L}}_{2,\tau}^\sigma, \overline{\hat{L}}_{1,\tau}^\sigma$.

The integral causal forcings (ICFs) presented in Table 3 are formalized in MJL using the corresponding empirical nomological statements to which they correspond.

We express in MJL all 14 integral causal forcings (ICF) contained in Table 3 and denoted by A_χ^σ, where $\chi \in \{a,b,c,d,e,f,g,h,i,j,k,l,m,n\}$. A_χ^σ are expressible through prolonged causal forcings A_r^σ, where r = 1, 2, 3, 4, 5, 6. In its turn, A_r^σ are expressible through six generators of hypotheses about causes and hypotheses about predictions that are defined for the corresponding histories of possible worlds HPW from $\overline{HPW}$.

Each prolonged CF (PCF) A_r^σ, where $1 \le r \le 6$, we express by $\exists h\tilde{A}_i^\sigma(h)$, where $\tilde{A}_i^\sigma(h)$ has the prefix $\forall V\forall Y\forall Z\forall p$. Thus, for example, we present A_1^σ as $\exists h\tilde{A}_1^\sigma(h)$, where $\tilde{A}_1^\sigma(h)$ have $\forall V\forall Y\forall Z\forall p((L_2^\sigma(V, Y, p, h)$ & $(V \subset Z)$ & $P(Z, p, h)) \rightarrow L_1^\sigma(Z, Y, p, h))$, and $\sigma = +, -$. Then, for each ICF from Table 3 we obtain the following representations of ICF, presented in Table 4.

ICF A_χ^σ, where $\chi = \{a, b, ..., m, n\}$, as characterized by the specification of their constituents PCF A_r^σ, where $1 \le r \le \sigma$ according to Table 3. The conditions for these specifications are the initial PCF, its descendants with the conditions $2q < s + 1$ and $2q \ge s + 1$ and conditions of monotony $\rho^\sigma(p)$ or their absence, which is indicated by M and $\neg M$, respectively [2].

By **X** and **Y** we will denote the variables for the elements of the set, which we denote by the names a, b, ..., m, n. By $\alpha(X)$, $\beta(X)$, $\gamma(X)$ and $\bar{M}(X)$ we will denote the conditions that characterize *ICF* A_χ^σ, where $\chi = \{a, b, ..., m, n\}$.

The values of $\alpha(\mathbf{X})$, $\beta(\mathbf{X})$ and $\gamma(\mathbf{X})$ are, respectively, $\langle v, v\rangle^*$, $\langle \tau, v\rangle^*$; $\langle v, v\rangle|\langle \tau, v\rangle$, $\langle \tau, v\rangle$; $2q < s + 1$, $2q \ge s + 1$. The values of $\bar{M}(x)$ are M, $\neg M$.

$\alpha(X)$ characterizes the initial PCF, $\beta(X)$ characterizes the immediate descendant of the initial PCF, $\gamma(\mathbf{X})$ characterizes the immediate descendant and all subsequent descendants, and $\bar{M}(x)$ characterizes the PCF itself.

On the set ER, we define the relation $\sqsupseteq$ by ordering $\alpha(X)$, $\beta(X)$, $\gamma(X)$, $\bar{M}(x)$ in the following way: $\langle v, v\rangle^* > \langle v, v\rangle|\langle \tau, v\rangle > \langle \tau, v\rangle$, $\langle v, v\rangle$ & $2q\langle s + 1\rangle > \langle v, v\rangle$ & $2q \ge s + 1$, $\langle \tau, v\rangle$ & $2q > \langle s + 1\rangle > \langle \tau, v\rangle$ & $2q \ge s + 1$; $\alpha(x) > \beta(x)$, $\langle v, v\rangle|\langle \tau, v\rangle > \langle \tau, v\rangle$, where ">" is the strict order relationship.

We define $x \sqsupset y$:

***Df.*13-3.** $x \sqsupset y$, if (1) or (2), or (3), or (4) occur:

(1) $\alpha(X) > \alpha(Y)$ & $\bar{M}(X) \ge \bar{M}(Y)$,

(2) $\alpha(X) = \alpha(Y)$ & $\beta(X) > \beta(Y)$ & $\bar{M}(X) \ge \bar{M}(Y)$,

(3) $\alpha(X) = \alpha(Y)$ & $\beta(X) = \beta(Y)$ & $\gamma(X) > \gamma(Y)$ & $\bar{M}(X) \ge \bar{M}(Y)$,

(4) $\alpha(\mathrm{X}) = \alpha(Y)$ & $\beta(X) = \beta(Y)$ & $\gamma(X) = \gamma(Y)$ & $\bar{M}(X) > \bar{M}(Y)$.

$X = Y$, if the corresponding conditions characterizing X and Y are equal. $X \sqsupseteq Y$, if $X \sqsupset Y$ or $X = Y$.

Proposal 2-3. The set of all integral causal forcings $\overline{ICF}$ is partially ordered and contains the largest and smallest elements.

Let $E = \overline{ICF}$, then $\overline{E} = \langle E, \sqsupseteq\rangle$ is a partially ordered set, where $E = \{a, b, c, d, e, f, g, h, i, j, k, l. m. n\}$ and $\forall x(a \sqsupseteq x)$ and $\forall x(x \sqsupseteq n)$.

Let $\psi_1(X)$, $\psi_2(X)$ and $\psi_3(X)$ be conditions (1), (2) and (3), respectively, then we define $X \sqsupseteq Y$: $\langle\psi_1(X) \vee \psi_2(X) \vee \psi_3(X), \bar{M}(X)\rangle \sqsupseteq \langle\psi_1(Y) \vee \psi_2(Y) \vee \psi_3(X), \bar{M}(X)\rangle \rightleftharpoons \bigvee_{i=}^{3}(\psi_1(X) \ge \psi_1(Y))$ & $(\bar{M}(X) \ge \bar{M}(Y))$.

Then, reflexivity occurs: $\forall x\ (x \sqsupseteq x)$, antisymmetry $\forall x\forall y\ (((x \sqsupseteq y)$ & $(y \sqsupseteq x)) \rightarrow x = y)$, as well as transitivity $\forall x\forall y\forall z(((x \sqsupseteq y)$ & $(y \sqsupseteq z)) \rightarrow (x \sqsupseteq z))$.

We consider the partition $E = E' \bigcup E''$, where $E' = \{a, c, e, g, i, k, m\}$, and $E'' = \{b, d, f, h, j, 1, n\}$ such that for all $x \in E$ there is $M(x)$, and for all $x \in E''$ there is $\neg M(x)$.

In E' and E'' there are, respectively, chains $a \sqsupset c$, $c \sqsupset e$, $e \sqsupset g$, $g \sqsupset i$, $i \sqsupset k$, $k \sqsupset m$; $b \sqsupset d$, $d \sqsupset f$, $f \sqsupset j$, $j \sqsupset l$, $l \sqsupset n$, which follows from the definitions A_χ^σ, where $\chi \in E$. Similarly, we have $a \sqsupset b$ and $m \sqsupset n$ (a and b, m and n are different by virtue of $\bar{M}(a) > \bar{M}(b)$ and $\bar{M}(m) > \bar{M}(n)$. Hence, $\forall x(a \sqsupseteq x)$ and $\forall x(x \sqsupseteq n)$.

The partially ordered set E represents $\overline{ICF}$, as $A_\chi^\sigma \in \overline{ICF}$ if and only if $\chi \in E$.

The set of integral forcings can be graphically represented by the classification Tree T (Fig. 1) as follows:

1^0. The root of the Tree T is $\overline{ICF}$ itself.

2^0. The branches $\mathrm{Br}(\chi)$ of the Tree T contain elements $A_\chi^\sigma \in \overline{ICF}$, where $\chi \in E$.

Table 3

PCF	Cd of initial PCF	Cd of descendants	Monotony $\rho^\sigma(p)$	ICF
$^{-}A_1^\sigma : L_2^\sigma, L_1^\sigma$ (L_2^σ, L_1^σ – generators ER)	$\langle v, v\rangle^*$	$\langle v, v\rangle$ & $(2q < s + 1)$, $\langle v, v\rangle \| \langle \tau, v\rangle$ & $(2q < s + 1)$	$\neg M$	A_b^σ
	$\langle v, v\rangle^*$	$\langle v, v\rangle$ & $(2q \geq s + 1)$, $\langle v, v\rangle$ & $(2q \geq s + 1) \| \langle v, v\rangle$ & $(2q < s + 1) \| \langle \tau, v\rangle$ & $(2q \geq s + 1) \|$ $\langle \tau, v\rangle$ & $(2q < s + 1)$	$\neg M$	A_d^σ
	$\langle v, v\rangle^*$	$\langle \tau, v\rangle$ & $(2q < s + 1)$	$\neg M$	A_f^σ
	$\langle v, v\rangle^*$	$\langle v, \tau\rangle$ & $(2q \geq s + 1)$, $\langle v, \tau\rangle$ & $(2q \geq s + 1) \|$ $\langle v, \tau\rangle$ & $(2q < s + 1)$	$\neg M$	A_h^σ
$A_2^\sigma : \hat{L}_2^\sigma, \hat{L}_1^\sigma$ ($\hat{L}_2^\sigma, \hat{L}_1^\sigma$ – generators ER)	$\langle v, v\rangle^*$	$\langle v, v\rangle$ & $(2q < s + 1)$, $\langle v, v\rangle \| \langle \tau, v\rangle$ & $(2q < s + 1)$	M	A_a^σ
	$\langle v, v\rangle^*$	$\langle v, v\rangle$ & $(2q \geq s + 1)$, $\langle v, v\rangle$ & $(2q \geq s + 1) \|$ $\langle v, v\rangle$ & $(2q < s + 1) \|$ $\langle \tau, v\rangle$ & $(2q \geq s + 1) \|$ $\langle \tau, v\rangle$ & $(2q < s + 1)$	M	A_c^σ
	$\langle v, v\rangle^*$	$\langle \tau, v\rangle$ & $(2q < s + 1)$	M	A_e^σ
	$\langle v, v\rangle^*$	$\langle v, \tau\rangle$ & $(2q \geq s + 1)$, $\langle v, \tau\rangle$ & $(2q \geq s + 1) \|$ $\langle v, \tau\rangle$ & $(2q < s + 1)$	M	A_g^σ
$A_3^\sigma : L_{2,\tau}^\sigma, L_{1,\tau}^\sigma$ ($L_{2,\tau}^\sigma, L_{1,\tau}^\sigma$ – generators ER)	$\langle \tau, v\rangle^*$	$\langle \tau, v\rangle$ & $(2q < s + 1)$	$\neg M$	A_j^σ
	$\langle \tau, v\rangle^*$	$\langle \tau, v\rangle$ & $(2q \geq s + 1)$, $\langle \tau, v\rangle$ & $(2q \geq s + 1) \| \langle \tau, v\rangle$ & $(2q < s + 1)$	$\neg M$	A_l^σ
$A_4^\sigma : \hat{L}_2^\sigma, \hat{L}_1^\sigma$ ($L_{2,\tau}^\sigma, L_{1,\tau}^\sigma$ – generators ER)	$\langle \tau, v\rangle^*$	$\langle \tau, v\rangle$ & $(2q < s + 1)$	M	A_i^σ
	$\langle \tau, v\rangle^*$	$\langle \tau, v\rangle$ & $(2q \geq s + 1)$, $\langle \tau, v\rangle$ & $(2q \geq s + 1) \|$ $\langle \tau, v\rangle$ & $(2q < s + 1)$	M	A_k^σ
$A_5^\sigma : \overline{L}_{2,\tau}^\sigma, \overline{L}_{1,\tau}^\sigma$	$\langle \tau, v\rangle^*$	$\langle \tau, v\rangle$ & $(2q \geq s + 1)$	$\neg M$	A_n^σ
$A_6^\sigma : \hat{L}_{2,\tau}^\sigma, \bar{\hat{L}}_{1,\tau}^\sigma$	$\langle \tau, v\rangle^*$	$\langle \tau, v\rangle$ & $(2q \geq s + 1)$	M	A_m^σ

3^0. The vertices of the Tree T are $\langle v, v\rangle^*$, $\langle \tau, v\rangle$; $\langle v, v\rangle|\langle \tau, v\rangle$, $\langle \tau, v\rangle$; $2q\langle s + 1\rangle$, $2q \geq s + 1$; M, $\neg M$.

4^0. M, $\neg M$ are the end vertices of the Tree T.

5^0. $\langle v, v\rangle^*$ and $\langle \tau, v\rangle^*$ directly follow the root $\overline{ICF}$.

6^0. The names of the branches of the Tree T are χ-elements $T\,E$: $\chi \in E$ [1, 2].

We denote by ER the set of element names $\overline{ICF}$, and EL, ET, SET, that is, designations of sets of names for ICF for empirical laws, empirical tendencies, and weak empirical tendencies, respectively:

$ER = \{a, b, c, d, e, f, g, h, i, j, k, l, m, n\}$,

$EL = \{a, b, c, d, e, f, g, h\}$,

$ET = \{i, j, k, l\}$,

$WET = \{m, n\}$.

It is obvious that $T = \{\mathrm{Br}(\chi)|\chi \in E\}$, and T one-to-one corresponds to $\overline{ICF}$.

<u>**Remark 9-3.**</u> We note that T is defined **independently** of $\overline{Str}$, i.e. for any $Str_{x,y}$.

<u>**Remark 10-3.**</u> We now replenish the initial conditions for applying the ASSR JSM method:

1^0. $\Omega(0, 1)$, $\Omega(1, 1)$, ..., $\Omega(s, 1)$, where $\Omega(0, 1) \subset \Omega(1, 1) \subset \ldots \subset \Omega(s, 1)$;

2^0. $\Omega^\tau(0, 1)$, where $\Omega^\tau(0, 1) = \Omega^\tau(p, h)$ for all p and all h;

3^0. $\overline{HPW}$;

4^0. $\overline{Str}$;

5^0. $\overline{ICF}$, where $\overline{ICF}$ represented by the Tree T.

Table 4

$$A_a^\sigma : \exists h_1(\tilde{A}_2^\sigma(h_1) \,\&\, \exists h_2(\tilde{A}_2^\sigma(h_2) \,\&\, \neg(h_1 = h_2)) \,\&\, \forall h(\neg(h = h_1) \to (\tilde{A}_2^\sigma(h) \vee \tilde{A}_4^\sigma(h))))$$

$$A_c^\sigma : \exists h_1(\tilde{A}_2^\sigma(h_1) \,\&\, \exists h_2(h_1 \neq h_2 \,\&\, \tilde{A}_2^\sigma(h_2)) \,\&\, \forall h((h \neq h_1) \,\&\, (h \neq h_2) \to \tilde{A}_2^\sigma(h) \mid \tilde{A}_4^\sigma(h) \mid \tilde{A}_6^\sigma(h)))$$

$$A_e^\sigma : \exists h_1(\tilde{A}_2^\sigma(h_1) \,\&\, \forall h((h \neq h_1) \to \tilde{A}_4^\sigma(h)))$$

$$A_g^\sigma : \exists h_1(\tilde{A}_2^\sigma(h_1) \,\&\, \exists h_2 \tilde{A}_6^\sigma(h_2) \,\&\, \forall h(h \neq h_1 \to (\tilde{A}_6^\sigma(h) \vee \tilde{A}_4^\sigma(h)))$$

$$A_6^\sigma : \exists h_1(\tilde{A}_1^\sigma(h_1) \,\&\, \forall h(\neg(h \neq h_1) \to (\tilde{A}_1^\sigma(h) \vee \tilde{A}_3^\sigma(h)))$$

$$A_d^\sigma : \exists h_1(\tilde{A}_1^\sigma(h_1) \,\&\, \exists h_2 \tilde{A}_5^\sigma(h_2) \,\&\, \forall h(h \neq h_1 \to (\tilde{A}_5^\sigma(h) \vee \tilde{A}_3^\sigma(h)))$$

$$A_f^\sigma : \exists h_1(\tilde{A}_1^\sigma(h_1) \,\&\, \forall h(h \neq h_1 \to \tilde{A}_3^\sigma(h))$$

$$A_h^\sigma : \exists h_1(\tilde{A}_1^\sigma(h_1) \,\&\, \exists h_2 \tilde{A}_5^\sigma(h_2) \,\&\, \forall h(h \neq h_1 \to (\tilde{A}_5^\sigma(h) \vee \tilde{A}_3^\sigma(h)))$$

$$A_i^\sigma : \exists h_1(\tilde{A}_4^\sigma(h_1) \,\&\, \forall h(h \neq h_1 \to \tilde{A}_4^\sigma(h))$$

$$A_k^\sigma : \exists h_1(\tilde{A}_4^\sigma(h_1) \,\&\, \exists h_2 \tilde{A}_6^\sigma(h_2) \,\&\, \forall h(\tilde{A}_6^\sigma(h) \vee \tilde{A}_4^\sigma(h)))$$

$$A_j^\sigma : \exists h_1(\tilde{A}_3^\sigma(h_1) \,\&\, \forall h(h \neq h_1 \to \tilde{A}_3^\sigma(h))$$

$$A_l^\sigma : \exists h_1(\tilde{A}_3^\sigma(h_1) \,\&\, \exists h_2 \tilde{A}_5^\sigma(h_2) \,\&\, \forall h(h \neq h_1 \to (\tilde{A}_5^\sigma(h) \vee \tilde{A}_3^\sigma(h)))$$

$$A_m^\sigma : \exists h_1(\tilde{A}_6^\sigma(h_1) \,\&\, \forall h(h \neq h_1 \to \tilde{A}_6^\sigma(h)))$$

$$A_n^\sigma : \exists h_1(\tilde{A}_5^\sigma(h_1) \,\&\, \forall h(h \neq h_1 \to \tilde{A}_5^\sigma(h)))$$

We recall that the vertices $\langle v, v\rangle^*$ and $\langle \tau, v\rangle^*$ immediately following the root of the Tree T are PCF of six types, that is, $A_1^\sigma, A_2^\sigma, A_3^\sigma, A_4^\sigma, A_5^\sigma, A_6^\sigma$, where $\sigma = +, -$, which correspond to the hypothesis generators L_2^σ-predicates and L_1^σ-predicates that are presented in Table 3.

The Tree T is generated by six generators of hypotheses of causes and hypotheses of predictions for $\overline{HPW}$.

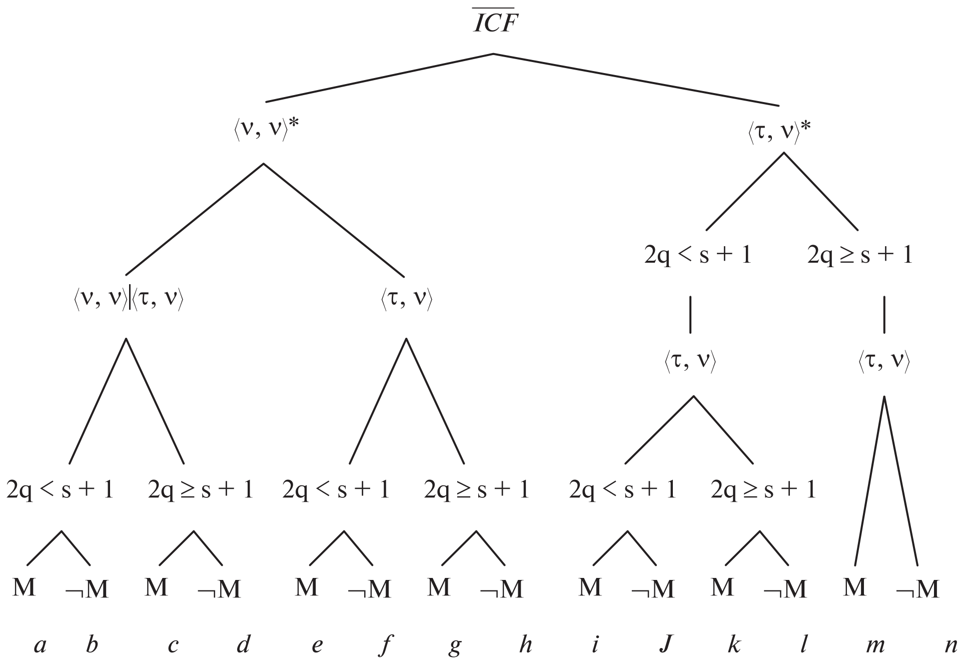

Fig. 1. A tree of T-classifications of empirical regularities.

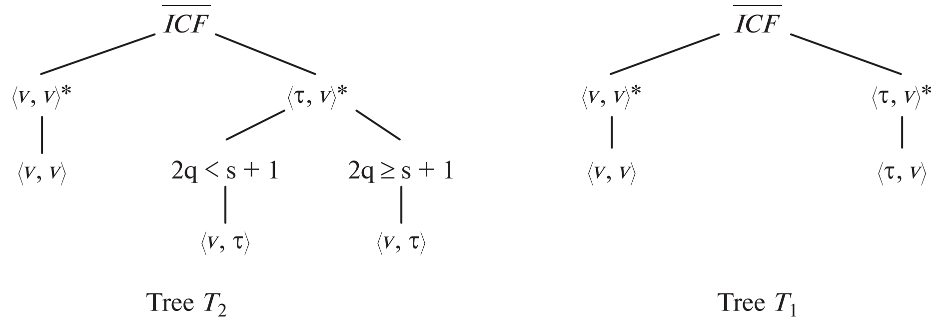

Fig. 2. Simplifications of the Tree T.

It is a general representation of all possible ICFs in the JSM research[8].

T can be transformed in two ways by canceling the conditions of M or $\neg M$, as well as the limitations and specifications of HPW from the set $\overline{HPW}$.

In the first case, only the condition M is preserved, while in the second, $\neg M$ is preserved. Then, we obtain, respectively, two Trees T' and T'', such that they are subtrees of T.

The Tree T has Br(χ) such that $\chi \in E'$, where $E' = \{a, c, e, g, i, k, m\}$, and T'' has Br(χ) such that $\chi \in E''$, where $E'' = \{b, d, f, h, j, l, n\}$, and $E = E' \bigcup E''$.

The Tree T''' is obtained by the transformation of T such that we omit both conditions M and $\neg M$. Then, the successors $\langle v, v\rangle \mid \langle v, \tau\rangle$, $\langle \tau, v\rangle$ will be $\langle \tau, v\rangle$, that is, end vertices, and by a, b; c, d; e, f; g, h; i, j; k, l; m, n, we replace, respectively, the names $ICF\ a_1, c_1, e_1, g_1, i_1, k_1$ and m_1.

T''' has $E''' = \{a_1, c_1, e_1, g_1, i_1, k_1, m_1\}$.

T', T'', T''' will be called **reduced trees** corresponding to $\overline{ICF}$.

We consider the second method of transforming the Tree T such that we change HPW in various ways.

Let us give two examples of such transformations of T.

(1) $\langle v, v\rangle^*$ has descendants only of the type $\langle v, v\rangle$, and $\langle \tau, v\rangle^*$ has descendants of the type $\langle \tau, v\rangle$ & $2q < s + 1$ and $\langle \tau, v\rangle$ & $2q \geq s + 1$.

(2) $\langle v, v\rangle^*$ has descendants only of the type $\langle v, v\rangle$, and $\langle \tau, v\rangle^*$ has descendants only of the type $\langle \tau, v\rangle$.

Thus, we have two simplifications of the Tree T (Fig. 2).

In T_2, the vertex $\langle v, v\rangle^*$ has descendants only of the type $\langle v, v\rangle$, and the vertex $\langle \tau, v\rangle^*$ has descendants either $\langle v, \tau\rangle$ & $2q < s + 1$, or $\langle v, \tau\rangle$ & $2q \geq s + 1$. In T_1, the vertex $\langle v, v\rangle$ * has only descendants of the type $\langle v, v\rangle$, and the vertex $\langle \tau, v\rangle^*$ has descendants of the type $\langle \tau, v\rangle$.

In §2, we considered the variant of JSM reasoning and JSM research for the simple case where the effect under study (this is the value of the variable Z) has a single cause (this is the value of the variable V) and, in addition, the JSM reasoning is realized in two steps: the use of p.i.r.-1 (induction) and the subsequent use of p.i.r.-2 (analogy). The use of p.i.r.-1 and p.i.r.-2 form **one clockcycle** of the JSM reasoning [6, 12]. This restriction was used for simple presentation by virtue of the fact that the general condition for the use of the ASSR JSM method will require only minor changes in its main characteristics. These changes are formulated below and concern the definitions of the generators $L_2^\sigma, L_1^\sigma; L_{2,\tau}^\sigma, L_{1,\tau}^\sigma; \overline{L}_{2,\tau}^\sigma, \overline{L}_{1,\tau}^\sigma$.

For generalized hypothesis generators, we retain the same notation, noting that instead of truth values $\langle v, 1\rangle$ for hypotheses about causes and truth values $\langle v, 2\rangle$ for hypotheses about predictions, we use, respectively, $\langle v, n\rangle$ and $\langle v, n + 1\rangle$, which are the truth values of the hypotheses obtained by using p.i.r.-1 and p.i.r.-2, respectively, where $n \geq 1$ and $v = 1, -1, 0$.

We suppose that the studied effect, which is the value of the variable Y, is represented by $Y = Y_1 \bigcup \ldots \bigcup Y_k$, and each Y_i corresponds to a possible cause, which is the value of the variable V_i, where $i = 1, \ldots, k$.

In accordance with these assumptions, we define hypothesis generators using the variable n, which denotes the number of applications of the plausible inference rules and expresses the likelihood of the generated hypotheses [6, 12], that is, the smaller n is, the greater the likelihood is.

For each V_i and the corresponding Y_i, we define L_2 generators and L_1 generators for empirical regularities such as laws, tendencies, and weak tendencies:

$L_2^+(V_i, Y_i, p, s, h) \rightleftharpoons \exists n_i\, (((0 \leq p \leq s)\ \&\ \rho^+(s) \geq \overline{\rho}^+)) \to (J_{\langle 1,n_i\rangle} H_2(V_i, Y_i, p, h) \in \tilde{\Delta}^+(p))\ \&\ (J_{\langle 1,1\rangle} H_2(V_i, Y_i, 0, h) \in \tilde{\Delta}^+(0)))$, where $1 \leq i \leq k$.

$L_1^+(Z, Y, p, s, h) \rightleftharpoons \exists n_i\, (((0 \leq p \leq s)\ \&\ \rho^+(s) \geq \overline{\rho}^+)) \to (J_{\langle 1,n_i\rangle} H_1(Z, Y, p, h) \in \tilde{\Omega}^+(p))\ \&\ (J_{\langle 1,2\rangle} H_1(Z, Y_i, 0, h) \in \tilde{\Omega}^+(0)))$.

[8] According to the terminology of I. Kant in "Critique of Pure Reason" [32], ICF are the conditions of "possible experience".

Similarly, we define $L_2^-(V_i, Y_i, p, S_i, h)$, $L_1^-(Z, Y, p, S_i, h)$, $L_{2,\tau}^{\sigma}(V_i, Y_i, p, S_i, h)$, $L_{1,\tau}^{\sigma}(Z, Y, p, S_i, h)$, $\overline{L}_{2,\tau}^{\sigma}(V_i, Y_i, p, S_i, h)$, $\overline{L}_{1,\tau}^{\sigma}(Z, Y, p, S_i, h)$, where $\sigma = +, -$.

We now define hypothesis generators about the causes and predictions, assuming that there may be k reasons for the subsets of the effect Y_i such that they have the corresponding causes V_i, and Y is a variable, whose value is the effect that is being studied, such that $Y = \bigcup_{i=1}^{k} Y_i$.

Thus, we believe that the studied effect Y has k reasons V_i, where $i = 1, \ldots, k$, such that any nonempty subset of it does not determine Y. Therefore, for any i, $1 \le i \le k$, $V_1, \ldots, V_{i-1}, V_{i+1}, V_k$ does not determine Y.

The above is formalized by Proposal 3-3, which expresses a prolonged causal forcing for k reasons and a possible iteration of the plausible inference rules in the JSM operator $O_{x,y}(\Omega(p))$ for $Str_{x,y}$. Proposal 3-2 is a generalization of Proposal 1-2.

Proposal 3-2. The conditions for prolonged causal forcing (PCF), determined by generators of hypotheses of causes and corresponding hypotheses of predictions, are true with respect to the histories of possible worlds $R^h(0)$, $R^h(1)$, ..., $R^h(s)$ for a fixed strategy $Str_{x,y}$ for the case of k causes of the effect under study and possible iterations of the plausible inference rules (p.i.r. 1 is induction and p.i.r. 2 is analogy).

Without loss of generality, we consider PCF for the positive reasons formulated below.

$\exists h \exists k \exists s_1 \ldots \exists s_k \forall V_1 \ldots \forall V_k \forall Y_1 \ldots \forall Y_k \forall Y \forall Z \forall p(((\&_{i=1}^{k}(L_2(V_i, Y_i, p, s_i, h) \,\&\, (\&_{i=1}^{k}(V_i \subset Z)) \,\&\, P(Z, p, h) \,\&\, P(Z, p, h) \,\&\, (Y = \bigcup_{i=1}^{k} Y_i)) \to L_1^+(Z, Y, p, s, h)) \,\&\, \exists s_0\, \exists p_0\, K^+(V_1, \ldots, V_{i-1}, V_{i+1}, \ldots, V_k, Y_1, \ldots, Y_{i-1}, Y_{i+1}, Y_k, \ldots Y, p_0, s_1, \ldots, s_{i-1}, s_{i+1}, s_k, h)$, where $K^+(V_1, \ldots, V_{i-1}, V_{i+1}, \ldots, V_k, Y_1, \ldots, Y_{i-1}, Y_{i+1}, Y_k, \ldots, Y, p_0, s_1, \ldots, s_{i-1}, s_{i+1}, \ldots, s_k, h) \rightleftharpoons ((L_2^+(V_1, Y_1, p_0, s_1, h) \,\&\, \ldots \,\&\, L_2^+(V_{i-1}, Y_{i-1}, p_0, s_{i-1}, h) \,\&\, L_2^+(V_{i+1}, Y_{i+1}, p_0, s_{i+1}, h) \,\&\, \ldots \,\&\, L_2^+(V_k, Y_k, p_0, s_k, h)) \to \neg L_1(Z, Y, p_0, s_0, h))$, where $s_0 = \max(V_1, \ldots, V_{i-1}, V_{i+1}, \ldots, V_k$.

The case of an effect determined by a set of reasons $V_1, \ldots, V_k$, where $k > 1$, such that it is preserved for a sequence of possible worlds $\Omega(p, h)$, is formalized by amplification p.i.r.-1. Similarly, the predicate for p.i.r.-1 needs to be amplified by the condition expressing that $V_1, \ldots, V_k$ is the smallest set of causes such that it determines the effect Y, which is preserved in the history of possible worlds $HPW\, R^h(0), R^h(1), \ldots, R^h(s)$. The amplified predicate $\tilde{P}_n^+(X, Y, p, h)$ used in the proof of Proposal 3-3 is defined below.

***Df.* 14-3**. $\tilde{P}_n^+(X, Y, p, h) \rightleftharpoons \exists k \exists V_1 \ldots \exists V_k \exists n_1 \ldots \exists n_k((((\&_{i=1}^{k} J_{\langle 1,n_i \rangle} H_2(V_i, Y_i, p, h) \,\&\, (Y = \bigcup_{i=1}^{k} Y_i)) \,\&\, \neg \exists V_0 \exists Y_0 \exists n_0\, ((J_{\langle -1,n_0 \rangle} H_2(V_0, Y_0, p, h) \vee J_{\langle 0,n_0 \rangle} H_2(V_0, Y_0, p, h)) \,\&\, (V_0 \subset X) \,\&\, (Y_0 \subset Y)))) \,\&\, \exists p_0\, (J_{\langle 1,n_1 \rangle} H_2(V_1, Y_1, p_0, h) \,\&\, \ldots \,\&\, J_{\langle 1,n_{i-1} \rangle} H_2(V_{i-1}, Y_{i-1}, p_0, h) \,\&\, J_{\langle 1,n_{i+1} \rangle} H_2(V_{i+1}, Y_{i+1}, p_0, h) \,\&\, \ldots \,\&\, H_2(V_k, Y_k, p_0, h)) \to \neg L_1^+(X, Y, p_0, h)))$.

Similarly, we define $\tilde{P}_n^-(X, Y, p, h)$.

Let the antecedent of the first subformula of the conjunction of Proposal 3-3 be true. Then, by the definition of $\tilde{P}_n^-(X, Y, p, h)$, the first conjunction subformula in the definition of $\tilde{P}_n^-(X, Y, p, h)$ is true. Since the subformula K^+ of Proposal 3-3 is true, the second subformula is true, expressing the fact that $V_1, \ldots, V_k$ is the smallest set of reasons that determine the effect Y. This proves the truth of the consequent and, therefore, the truth of PCF for A_1^+, A_2^+ and A_1^-, A_2^-. Similarly, the truth of PCF is established for A_i^{σ}, where $i = 3, 4, 5, 6$, presented in Table 3.

Proposal 3-3 and its extension to the case A_i^{σ} extends the content of the Tree T representing the *ICF*.

4. REALIZATIONS OF INTEGRAL CAUSAL FORCINGS: DEFINITION OF A SET OF EMPIRICAL REGULARITIES AND RELATED MODALITIES

From Proposals 1-2 and 3-3 it follows that in MJL empirical pre-regularities A_i^{σ}, where $1 \le i \le \sigma$, $\sigma = +,-$ regarding some histories of possible worlds HPW_h, $h = 1, \ldots, (s + 1)!$: $v[A_i^{\sigma}] = t$, are true.

Since Proposal 3-3 generalizes Proposal 1-2 for the case of the set of causes $V_1, \ldots, V_k$, where $k \ge 1$, then A_i^{σ} can represent CF for empirical pre-regularities formed by k causes of the effect Y.

There are six CF options, corresponding to *PEL*, *PET*, and *PWET*, that is, types of empirical pre-regularities expressed by $\exists h \exists s \forall V \forall Y \forall Z \forall p\, ((D_{2,j}^{\sigma}(V, Y, Z, p, s, h) \to D_{1,j}^{\sigma}(Z, Y, p, s, h))$, where $1 \le i \le \sigma$, $\sigma = +, -$. It was shown above that in MJL from the truth of the antecedent $D_{2,j}^{\sigma}(V, Y, Z, p, s, h)$ follows the truth of the consequent $D_{1,j}^{\sigma}(Z, Y, p, s, h)$.

We note that according to Df. 12-2, formulating the condition for applicability of the ASSR JSM method, there exist Str_{x_1,y_1} or Str_{x_2,y_2} such that $\neg(D_{2,j}^+ = \Lambda) \vee \neg(D_{2,i}^- = \Lambda)$, where $1 \le i, j \le \sigma$. This assumption is necessary for the existence of empirical regularities.

Under conditions of CF for empirical pre- regularities (*PEL*, *PET*, *PWET*), we replace the variables *V*, *Y*, *h*, and *s* with the corresponding constants *C*', *Q*, $\overline{h}$, $\overline{s}$ and obtain $\forall Z \forall p((D^{\sigma}_{2,j}\ (C', Q, Z, p, \overline{s}, \overline{h}) \to D^{\sigma}_{1,j}(Z,Y, p, \overline{s}, \overline{h}))$, which we denote by $A^{\sigma}_{j}(C',Q, \overline{s}, \overline{h})$.

***Df.*15-4.** $A^{\sigma}_{j}(C', Q, \overline{h})$, where $1 \le j \le \sigma$, will be called the **realization** of the empirical pre-regularity (PER) generated by prolonged causal forcing (*PCF*)A^{σ}_{j}, where $\sigma = +, -$.

Thus, the pair $\langle C',Q\rangle$, representing the cause and effect, performs CF A^{σ}_{j}. We will denote the realization of the prolonged CF by *RPCF.*

Table 3 shows the dependences of the integral causal coercion (*ICF*) on the corresponding generators of hypotheses about causes and hypotheses about predictions that generate the *ICF.* Thus, the set of integral causal forcings is formed by the corresponding set of PCFs. The set of all ICFs is denoted by $\overline{ICF}$.

Obviously, the Tree *T* graphically represents the set $\overline{ICF}$ such that each branch Br(χ) conform to ICFs of the type χ, where $\chi \in \{a, b, c, d, e, f, g, h, i, j, k, l. m. n\}$.

We recall that *T* is defined independently of the set of strategies $\overline{Str}$ JSM reasoning.

Thus, there is a one-to-one correspondence between $T = \{\mathrm{Br}(\chi) | \chi \in E\}$ and the set $\overline{ICF}$ of all integral causal forcings.

The set $\overline{ICF}$, therefore, is representable as follows: $\overline{ICF} = \{A^{\sigma}_{a}, A^{\sigma}_{b}, A^{\sigma}_{c}, A^{\sigma}_{d}, A^{\sigma}_{e}, A^{\sigma}_{f}, A^{\sigma}_{g}, A^{\sigma}_{h}, A^{\sigma}_{i}, A^{\sigma}_{j}, A^{\sigma}_{k}, A^{\sigma}_{m}, A^{\sigma}_{n}\}$, where $\sigma = +, -$, which is expressed in Table 3. We consider A^{σ}_{χ}, $\chi \in E$, $\sigma = +, -$, where A^{σ}_{χ} characterizes one-to-one Br(χ) in *T.* We will use the JSM reasoning for all *HPW* from $\overline{HPW}$, realizing JSM research [1, 2]. Therefore, we will apply all $Str_{x,y}$ out of many strategies $\overline{Str}$. This means the generation of multiple realizations of $A^{\sigma}_{j}(C',Q, \overline{s}, \overline{h})$ according to *Df.*15-4.

***Df.*16-4**. The set of PCF realizations that form the Br(χ) of the tree *T* will be called the realization of integral causal forcing A^{σ}_{χ}, where $\chi \in E$. Realization of *ICF* A^{σ}_{χ} will be denoted by $A^{\sigma}_{\chi}(C',Q)$, where $\langle C',Q\rangle$ forms realization A^{σ}_{χ}, *C*' is the value *V*, and *Q* is the value *Z*.

Realizations of the ICFs will be denoted by RICF. The set of all realizations of the ICFs is denoted by $\overline{RICF}$. Since realizations of the ICFs are generated by strategies $Str_{x,y}$ of JSM reasoning, where $x \in I^{+}$, $y \in I^{-}$ [6, 13]; $Str_{x,y} \in \overline{Str}$, and $\overline{Str}$ is a distributive lattice.

Empirical regularity will be called the realization of integral causal forcing RICF. The set of all realizations of the ICFs is denoted by $\overline{RICF}$.

Now let us clarify the idea of empirical regularity. The set of all empirical regularities previously denoted by *ER* is $\overline{RICF}$. In this way, $ER = \overline{RICF}$.

Previously, it was believed that $ER = \{a, b, c, d, e, f, g, h, i, j, k, l, m, n\}$, but now we will understand $\chi \in$ ER as **types** of empirical regularities and we denote the realizations χ by $\overline{\chi}$. In this way, $\overline{ER} = \{\overline{a}, \overline{b}, \ldots, \overline{m}, \overline{n}\}$ are the set of all empirical regularities, where $\overline{\chi}$ is the realization A^{σ}_{χ}, and $\chi \in ER$, $\sigma = +, -$; $\overline{ER} = \overline{EL} \cup \overline{ET} \cup \overline{WET}$.

<u>**Remark 11-4.**</u> $\overline{ER}$ set of **types** of realizations of empirical regularities $\overline{\chi}$. It means that $\overline{\chi}$ is a **factor set** of specific regularities of the type $\overline{\chi}$, where A^{σ}_{χ} corresponds to the *ICF,* and $\overline{\chi} = \{\tilde{\chi}_1, \ldots, \tilde{\chi}_l\}$, where $\overline{\chi}_i$, $1 \le i \le l$, is the empirical regularity formed by the pair $\langle C'_i, Q_i\rangle$, where C'_i and Q_i present hypotheses about the cause and the corresponding effect. The set of all specific empirical regularities is denoted by $\overline{\overline{ER}}$. Empirical regularities corresponding to pairs $\langle C'_i, Q_i\rangle$ and $\langle C'_j, Q_j\rangle$, will be called **similar** if they are generated by the ICF A^{σ}_{χ}. Such empirical laws will be called **equivalent** if they have the same effect.

Thus, there are three sets that characterize empirical regularities, that is, *ER*, $\overline{ER}$, and $\overline{\overline{ER}}$. *ER* forms an **intensional** of the concept of the set of "empirical regularities" (*IntER*), and $\overline{ER}$ and $\overline{\overline{ER}}$ represent an **extensional** (*ExtER*) of this concept of the ASSR JSM method. More precisely, $ExtER = \overline{\overline{ER}}$, and $\overline{ER}$ is the factor set for elements $\overline{\overline{ER}}$[9].

Table 3 shows all 14 types of ICF that make up $\overline{ICF}$, $\overline{ICF} = ER$ is an intensional of the concept "set of empirical regularities," as formulated above. Empirical regularities themselves are elements of $\overline{\overline{ER}}$, where $\overline{\overline{ER}} = Ext\ ER$. Therefore, it is correct that $ER = Int\overline{\overline{ER}}$. In this way, $Int\overline{\overline{ER}} = \overline{ICF}$, that is, the set of integral forcings represented by the tree *T* and described in Table 3.

Table 3 is based on the semantics of a **finite** set of histories of **finite** possible worlds, which are FB(*p*, *h*), one-to-one corresponding to $\Omega(p, h)$, where $p = 0, 1, \ldots, s$, and the domain *h* is a set of histories of possible worlds $\overline{HPW}$, where $|\overline{HPW}| = (s + 1)!$[10].

---

[9] In [3], *Int* and *Ext* were considered for the initial predicates of the *JSM* method and the plausible inference rules.

[10] For simplicity, we will use the number *i* instead of HPW_i.

Table 3 presents a characterization of all ICF A_χ^σ, where $\chi = \{a, b, ..., m, n\}$, and $\sigma = +, -$. In this way, A_χ^σ are elements of $Int\overline{\overline{ER}} = \overline{JCF}$. However, A_χ^σ are determined by generators $L_2^\sigma, L_1^\sigma; \hat{L}_2^\sigma, \hat{L}_1^\sigma; L_{2,\tau}^\sigma, L_{1,\tau}^\sigma; \hat{L}_{2,\tau}^\sigma, \hat{L}_{1,\tau}^\sigma; \overline{L}_{2,\tau}^\sigma, \overline{L}_{1,\tau}^\sigma; \overline{\hat{L}}_{2,\tau}^\sigma, \overline{\hat{L}}_{1,\tau}^\sigma$, generating both **initial** PCFs with codes $\langle v, v\rangle^*$ and $\langle \tau, v\rangle^*$, and their descendants characterized by the conditions $2q < s + 1$, $2q \geq s + 1$ and M, $\neg M$, which is presented in the classification tree T of ICF, the root of which is $\overline{ICF} = Int\overline{\overline{ER}} = ER$.

JCF there is an element of **content** of the intensional, and a partially ordered set $\langle E, \sqsupseteq\rangle$, where $E = \overline{ICF}$, is the **ordering** of content $Int\overline{\overline{ER}}$.

We consider realizations of integral causal forcings RICFs for the set $\overline{ICF}$, which in accordance with Table 3, Tree T and definitions A_χ^σ, where $\chi \in E$, and $E = E' \cup E'' \cup E'''$, $E' = \{a, b, c, d, e, f, g, h\}$, $E'' = \{i, j, k, l\}$, $E''' = \{m, n\}$.

***Df.*17-4.** We define modal operators $\Box\chi_1, \Diamond\chi_2, \nabla\chi_3$ of necessity, possibility, and weak possibility for realizations A_i^σ and A_χ^σ, where $1 \leq i \leq 6$, $\chi \in E$, $\sigma = +, -$:

$\Box\chi_1 A_i^\sigma(C', Q) \rightleftharpoons A_{\chi_1}^\sigma(C', Q)$, where $\chi_1 \in \{b, d, f, h\}$;

$\Box\chi_2 A_2^\sigma(C', Q) \rightleftharpoons A_{\chi_2}^\sigma(C', Q)$, where $\chi_2 \in \{a, c, e, g\}$;

$\Diamond\chi_3 A_3^\sigma(C', Q) \rightleftharpoons A_{\chi_3}^\sigma(C', Q)$, where $\chi_3 \in \{j, l\}$;

$\Diamond\chi_4 A_4^\sigma(C', Q) \rightleftharpoons A_{\chi_4}^\sigma(C', Q)$, where $\chi_4 \in \{i, k\}$;

$\nabla\chi_5 A_5^\sigma(C', Q) \rightleftharpoons A_{\chi_5}^\sigma(C', Q)$, where $\chi_5 = n$;

$\nabla\chi_6 A_6^\sigma(C', Q) \rightleftharpoons A_{\chi_6}^\sigma(C', Q)$, where $\chi_6 = m$.

Earlier in §2, PCF A_r^σ, where $1 \leq r \leq 6$ were expressed through $\exists h \tilde{A}_r^\sigma(h)$, where $\tilde{A}_r^\sigma(h)$ has a prefix $\exists s \forall V \forall Y \forall Z \forall p$; then, for example, A_i^σ we represent as $\exists h \tilde{A}_r^\sigma(h)$, where $\tilde{A}_1^\sigma(h) \rightleftharpoons \exists s \forall V \forall Y \forall Z \forall p((L_2^\sigma(V, Y, p, s, h)\ \&\ (V \subset Z)\ \&\ P(Z, p, h)) \to L_1^\sigma(Z, Y, p, s, h))$, and $\sigma = +, -$.

Similarly, we represent PCF A_r^σ, where $1 \leq r \leq 6$.

Replacing V and Y, respectively, with constants C' and Q, we obtain the realizations RPCF A_r^σ— $\exists h \tilde{A}_r^\sigma(h, C', Q)$, where $1 \leq r \leq 6$. Then, we define assessments of statements with modal operators $\Box\chi$, $\Diamond\chi, \nabla\chi$, representing realizations ICF as follows: $V[\Box_b A_1^\sigma[C', Q] = t$, if there exists HPW h_1 such that $V[\tilde{A}_1^\sigma(C', Q, h_1)] = t$ and for all HPW h such that if $\neg(h = h_1)$, then $V[\tilde{A}_1^\sigma(C', Q, h)] = t$ or $V[\tilde{A}_3^\sigma(C', Q, h)] = t$;

$V[\Box_a A_2^\sigma(C', Q)] = t$, if there exists HPW h_1 such that $V[\tilde{A}_2^\sigma(C', Q, h_1)] = t$ and there exists HPW h_2 such that $\neg(h_1 = h_2)$ and $V[\tilde{A}_2^\sigma(C', Q, h_2)] = t$ and for all HPW h such that if $\neg(h = h_1)$, then $V[\tilde{A}_2^\sigma(C', Q, h)] = t$ or $V[\tilde{A}_4^\sigma(C', Q, h)] = t$; similarly, we define $V[\varphi]$ for $\chi = c, d, e, f, g, h$.

$V[\Diamond_i A^\sigma(C', Q)] = t$, if there exists HPW h_1 such that $V[\tilde{A}_4^\sigma(C', Q, h_1)] = t$ and for all HPW h such that if $\neg(h = h_1)$, then $V[\tilde{A}_4^\sigma(C', Q, h)] = t$; similarly, we define $V[\varphi]$ for $\chi = i, j, k, l$.

$V[\nabla_n A_5^\sigma(C', Q)] = t$, if there exists HPW h_1 such that $V[\tilde{A}_5^\sigma(C', Q, h_1)] = t$ and for all HPW h such that if $\neg(h = h_1)$, то $V[\tilde{A}_5^\sigma(C', Q, h)] = t$;

$V[\nabla_m A_6^\sigma(C', Q)] = t$, if there exists HPW h_1 such that $V[\tilde{A}_6^\sigma(C', Q, h_1)] = t$ and for all HPW h such that if $\neg(h = h_1)$, then $V[\tilde{A}_6^\sigma(C', Q, h)] = t$.

Thus, RICFs are realizations of integral causal forcings defined for all HPW from $\overline{HPW}$, and are **empirical nomological statements**.

We note that in [30, 31] H. Reichenbach proposed the theory of nomological statements that expressed both the laws of logic and the laws of nature. The empirical nomological statements defined above satisfy the **applicability** condition for the ASSR JSM method formulated in *Df.*12-2. *Df.*17-4 and represent empirical nomological statements as modal statements.

The set $\overline{Str}$ corresponds to four direct products of lattices of intensionals of M-predicates $IntL^+ \times \neg\ IntL^-$, $Int\neg\ L^+ \times IntL^-$, $IntL^+ \times IntL^-$, $Int\neg\ L^+ \times Int\neg\ L^-$ [6, 13]. To consider empirical regularities, one should use $IntL^+ \times \neg\ IntL$ and $Int\neg\ L^+ \times IntL^-$, respectively for ER^+, $\overline{\overline{ER}}^+$ and ER^-, $\overline{\overline{ER}}^-$. $Int(M_{x,n}^+(V, Y)\ \&\ \neg M_{y,n}^-(V, Y)) = M_{x,y}^+(V, Y)\ \&\ \neg M_{y,n}^-(V, Y)$, $Int(\neg M_{x,n}^+(V, Y)\ \&\ M_{y,n}^-(V, Y))$; $Ext(M_{x,n}^+(V, Y)\ \&\ \neg M_{y,n}^-(V, Y)) = \{\langle V, Y\rangle \mid M_x^+(V, Y)\}$, $Ext(\neg M_{x,n}^+(V, Y)\ \&\ M_{y,n}^-(V, Y)) = \{\langle V, Y\rangle \mid \neg M_{x,n}^+(V, Y)\ \&\ M_{y,n}^-(V, Y)\}$, where $x \in I^+$, $y \in I^-$, and I^σ — the set of names of M^σ-predicates ($\sigma = +, -$) [3].

Int and *Ext* of the plausible inference rules of the first kind (induction) correspond to the products of distributive lattices of M^σ-predicates and their negations [6, 13].

In this way, $Str_{x,y}$ of JSM reasoning is formed by p.i.r.-1 $(I)_{x,y}^\sigma$ and p.i.r.-2 $(II)_{x,y}^\sigma$, where $\sigma = +, -$.

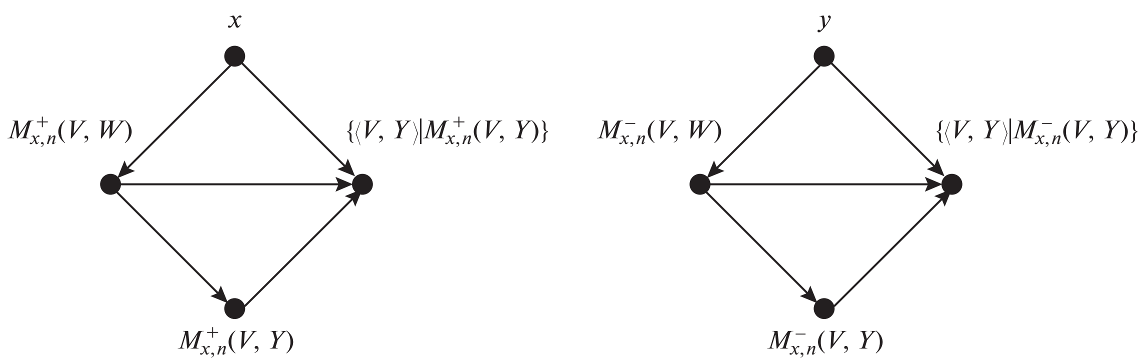

Fig. 3. @

In [6], it was shown that p.i.r.-1 is partially ordered by the relation $\succcurlyeq : \langle I^+ \times \neg I^-, \succcurlyeq\rangle$ and $\langle \neg I^- \times I^-, \succcurlyeq\rangle$. These partially ordered sets correspond to $(I)^+_{x,y}$ and $(I)^-_{x,y}$ for $Str_{x,y}$. This partial order was also preserved for p.i.r.-2 $(I)^\sigma_{x,y}$, which means partial ordering of $Str_{x,y}$, generated by p.i.r.-1 and p.i.r.-2, realized in $\bar{O}_{x,y}(\Omega(p,h))$.

It is significant to note that $Str_{x,y}$ is a method of realization of $Int\overline{\overline{ER}}$, forming the **procedures** for obtaining $ExtER = \overline{\overline{ER}}$.

In [3], a change in the G. Frege triangle was proposed to represent procedural concepts. M^σ-predicates that form p.i.r.-1 (as conceptual constructions) that are representable using quadrangles with vertices x, y are names; $M^+_{x,n}(V,Y), M^-_{y,n}(V,Y)$ are intensionals; $\{\langle V,Y\rangle \mid M^+_{x,n}(V,Y)\}$, $\{\langle V,Y\rangle \mid M^-_{y,n}(V,Y)\}$ are extensionals; $[M^+_{x,n}(V,Y)]$, $[M^-_{y,n}(V,Y)]$ are procedural expressions such that for each $\langle V, Y\rangle$ extensionals are formulated, that is, a condition for the truth of intensionals (Fig. 3).

Similarly, they are defined for p.i.r.-2$(II)^\sigma_{x,y}$, where $\sigma = +, -$ (Fig. 3a).

Procedural expressions carry out the **constructivization** of the intensional, generating an extensional, which is a feature of the procedural concepts represented by the triple $\langle Int, PrInt, Ext\rangle$, where $PrInt$ is the procedural expression that formulates the method for generating Ext. In the ASSR JSM method, algorithms for generating similarities of facts are used to perform induction [33, 34].

In [3], schemes for representing procedural concepts for p.i.r.-1 were also formulated.

The "set of empirical regularities" concept is an important example of a procedural concept, whose representation scheme is given below, that is, it expresses the organization of the triple $\langle Int\overline{\overline{ER}}$, $PrInt\overline{\overline{ER}}$, $Ext\overline{\overline{ER}}\rangle$, where $PrInt\overline{\overline{ER}}$ is realized through the set $\overline{Str}$-strategies of *JSM* reasoning.

Thus, $PrInt\overline{\overline{ER}}$ is formed by the set of strategies of JSM reasoning $\overline{Str}$ [6, 13]. In [13], two cases of $\overline{Str}$ are considered: with 16 and 36 strategies of JSM reasoning[11]. In the first case, p.i.r.-1 are formulated using four positive and four negative M^σ-predicates ($\sigma = +, -$) $M^+_{x,n}$, where x is a^+, $(ab)^+$, $(ad_0)^+$, $(ad_0b)^+$, and $M^-_{y,n}$, where y is a^-, $(ab)^-$, $(ad_0)^-$, $(ad_0b)^-$ [6, 13]; and a^σ, $(ab)^\sigma$, $(ad_0)^\sigma$, $(ad_0b)^\sigma$ are conditions for M^σ-predicates of similarity, similarities with prohibition of counterexamples, difference and differences with prohibition of counterexamples, respectively[12].

$Int\overline{\overline{ER}}$ we represent as a tree $T = \{\mathrm{Br}(\chi) | \chi \in E\}$, $PrInt\overline{\overline{ER}}$ we represent as the set of all strategies of *JSM* reasoning [6, 13] $\overline{Str}$, where $\overline{Str} = \{Str_{x_1,y_1}, \ldots, Str_{x_\rho,y_\rho}\}$, where $x_i \in I^+$, $y_i \in I^-$, $1 \le i \le \rho$, and $\rho = 16, 36$ [6, 13].

To each branch $\mathrm{Br}(\chi)$ of the tree T we assign a set of pairs $\langle C_r', Q_r\rangle^\sigma$, where $\sigma = +, -$, such that they represent the cause and corresponding effect of some empirical regularity from the generated set $\overline{\overline{ER}}$ through some $Str_{x,y}$ from $\overline{Str}$.

We introduce the following notations for this purpose: $\mathcal{A}^\sigma_{x,y} = \bigcup_{\chi \in E} \{\langle V,Y\rangle \mid A^\sigma_\chi(V,Y)\}$, where $\sigma = +, -$; $A^\sigma_\chi(V,Y)$ represents realizations ICF for $Str_{x,y}$ from $PrInt\overline{\overline{ER}}$.

[11] In [35], a description is given of an intelligent system that implements the ASSR JSM method for gastroenterology data. This computer system has 16 JSM strategies.

[12] Conditions a and ad_0 formalize inductive canons of similarity and difference [14]. The canons of similarity-differences are formalized in [13, 36].

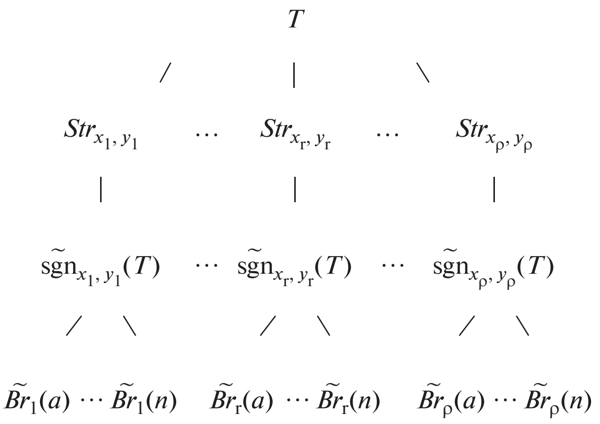

Fig. 4. Scheme *Concept ER* ($\widetilde{\mathrm{Br}}_r(\chi)$ is the labeled or unlabeled branch $\mathrm{Br}_r(\chi)$, generated by Str_{x_r,y_r})

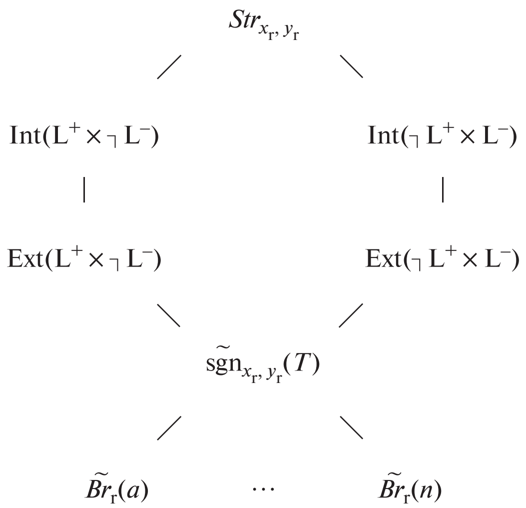

Fig. 5. Specification and complication of the Scheme *Concept ER*.

Branch $\mathrm{Br}(\chi)$ with attributed $\mathscr{A}_{x,y} = \mathscr{A}^+_{x,y} \cup \mathscr{A}^-_{x,y}$ will be denoted by $\mathrm{Br}(\chi)|A_{x,y}$ and called the **labeled branch**: $\mathrm{sgn}_{x,y}\mathrm{Br}(\chi) \rightleftharpoons \mathrm{Br}(\chi)|A_{x,y}$, where $\chi \in E$.

A tree $\mathrm{sgn}(T)$ will be called **labeled** if it consists of branches such that some of them are labeled, i.e., there are $\mathrm{sgn}_{x,y}\mathrm{Br}(\chi)$.

Through $\widetilde{\mathrm{sgn}}\,\mathrm{Br}(\chi)$ we denote the branch $\mathrm{Br}(\chi)$ such that it is labeled or unlabeled.

We obtain the following scheme of the procedural concept "set of empirical regularities" *Concept* $ER = \langle Int\overline{\overline{ER}}, PrInt\overline{\overline{ER}}, ExtER\rangle$: (Fig. 4).

The scheme *Concept ER* should be complicated by representing it as a concept that has *Int* and *Ext* for p.i.r.-1 (induction) and p.i.r.-2 (analogy), forming a JSM reasoning with $\bar{O}_{x,y}(\Omega(p))$ and $\rho^\sigma(p)$, where $\sigma = +, -$.

Since p.i.r.-2 is determined by the results of applying p.i.r.-1, we supplement the Scheme *Concept ER* with a simplification by adding only Ext and Int for p.i.r.-1 (inductive inference rules) [13].

In [13], distributive lattices for p.i.r.-1 $(I)^\sigma_{x,y}$ were considered, where $\sigma = +, -, 0, \tau$. Since for definition of empirical regularities it is enough to use $(I)^\sigma_{x,y}$ with $\sigma = +, -$, then for their representation we will use $Int(L^+ \times \neg\, L^-)$, $Ext(L^+ \times \neg\, L^-)$ and $Int(\neg\, L^+ \times L^-)$, $Ext(\neg\, L^+ \times L^-)$, respectively. Then, we add the following constructions for Str_{x_r,y_r}, to the Scheme *Concept ER*, where $r = 1, \ldots, \rho$ (Fig. 5).

We note that the corresponding Int and Ext of p.i.r.-1 are distributive lattices [13].

Concept ER consists of:

content $Int\overline{\overline{ER}}$ $ER = \{A^+_a, A^+_b, \ldots, A^+_m, A^+_n\} \cup \{A^-_a, A^-_b, \ldots, A^-_m, A^-_n\}$, **ordering** content through $\bar{E} = \langle E, \sqsupseteq\rangle$, where $E = \{a, b, \ldots, m, n\}$ is a set of **types** of empirical regularities; **constructivization** [3] $Int\overline{\overline{ER}}$ – $PrInt\overline{\overline{ER}} = \left\{\widetilde{\mathrm{sgn}}_{x_1y_1}(T), \ldots, \widetilde{\mathrm{sgn}}_{x_\rho y_\rho}(T)\right\}$ for all $Str_{x,y}$ from $\overline{Str}$; **extensional** $ExtER_{x,y} = \left(\bigcup_{x\in\bar{E}^+}\{\langle V,Y\rangle | A^+_\chi(V,Y)\} \cup \left(\bigcup_{x\in\bar{E}^-}\{\langle V,Y\rangle | A^-_\chi(V,Y)\}\right)\right)$, $ExtER = \{ExtER_{x,y} \mid Str_{x,y} \in \overline{Str}\}$.

Thus, **procedural concepts** consist of intensional (content, its ordering and constructivization) and extensional generated by constructivization of intensional. This structure of procedural concepts is subordinated to the main principle of semiotics: **the extensional is the function of the intensional** [36].

Remark 12-4. $PrInt\overline{\overline{ER}}$ is a partially ordered set, the partial order of which is generated by the partial order on $\overline{Str}$ [6].

Thus, the structure of Concept *ER* has **two partial orders**, that is, for a set of types of regularities E and for the set $\overline{Str}$.

Remark 13-4. The extension of the "set of empirical regularities" concept is represented by empirical nomological statements with modal operators, since $A^\sigma_{\chi_1}(C',Q)$ is $\square_{\chi_1} A^\sigma_1(C',Q)$, where $\chi_1 \in \{b,d,f,h\}$, $A^\sigma_{\chi_2}(C',Q)$ is $\square_{\chi_2} A^\sigma_2(C',Q)$, where $\chi_2 \in \{a,c,e,g\}$.

Similarly, $A^\sigma_{\chi_3}$ and $A^\sigma_{\chi_4}$ are representable through $\diamond_{\chi_3}$ and $\diamond_{\chi_4}$; and $A^\sigma_{\chi_5}$ and $A^\sigma_{\chi_6}$ are representable through ∇_{χ_5}, ∇_{χ_5}, respectively.

Since pairs $\langle C', Q\rangle$ "cause–effect" form Ext ER, if they realize ICF, then for the initial data of the JSM research [1, 2], the number of possible empirical regularities $\lambda_0 = m_0(2^{r^0} - 1)$, is definable, where $m_0 = |\Omega^\tau(0)|$, and $r_0 = \left|\left(\bigcup_{\chi\in \bar{\bar{E}}^+}\{\langle V, Y\rangle \mid A^+_\chi(V, Y)\}\right) \cup \left(\bigcup_{\chi\in \bar{\bar{E}}^-}\{\langle V, Y\rangle \mid A^-_\chi(V, Y)\}\right)\right|$ takes the fact into account that the effect can have many causes $V_1, \ldots, V_k$.

We now define possible varieties of *JSM* research using the *ExtER* specification using the Scheme *Concept ER*.

We say that the branch Br(χ) of the Tree sgn(T) is **dry** if it is unlabeled. Then, T itself is formed by dry Br(χ), if it consists only of dry branches.

Df. 18-4. Definition of *ExtER* specifications.

(1) A Tree sgn(T) will be called **healthy** if it does not have dry branches.

(2) A labeled Tree sgn(T) will be called sick if it has dry branches.

(3) A set of trees corresponding to $PrInt\bar{\bar{E}}$, i.e., generated by application of *ICF* for $\overline{HPW}$, is called a **forest** formed by $\tilde{\text{sgn}}(T)$ for all $Str_{x,y}$ from $\overline{Str}$.

(4) A forest is called **complete** if for each $Str_x \in \overline{Sstr}$, there is a labeled tree $\text{sgn}_{x_r, y_r}(T)$, where $1 \le r \le \rho$.

(5) A complete forest is called **thick** if it consists only of healthy trees.

(6) A thick forest is called **impenetrable** if the branches sgn((χ), $1 \le r \le \rho$, of all its trees are generated by *ICF* according to Proposal 2-2, that is, they are labeled with a **nonsingular** set of "cause–effect" pairs.

According to the Scheme Concept *ER* and *Df.* 18-4, $Int\bar{\bar{\bar{ER}}}$ has the variety of *ExtER* corresponding to the forest species generated by $Pr\ Int\bar{\bar{\bar{ER}}}$. Specific ExtERs will be called **exemplifications** of $Int\bar{\bar{\bar{ER}}}$. These exemplifications differ in the content of intensionality, the presence or absence of dry branches, and the singularity or nonsingularity of labeled branches. Exemplifications of $Int\bar{\bar{\bar{ER}}}$ for the conducted JSM research form a domain **modeling** by $\overline{Str}$, $\overline{HPW}$ и $\overline{ICF}$ for the initial data of the *JSM* research.

The last step in the application of the ASSR *JSM* method is *JSM* research [1, 2], whose purpose is the formation and expansion of quasi-axiomatic theories (QATs) [2, 4, 12]. QATs are a means of representing knowledge and their organization in the knowledge bases of intelligent systems that perform the ASSR *JSM* method.

QATs are **open** theories with expandable arrays of facts *FB*(p), where $p = 0, 1, \ldots, s$, logical means of which are *JSM* reasoning used in *JSM* research.

QATs are defined for fixed strategies of *JSM* reasoning $Str_{x,y}$ from the set $\overline{Str}$ of set strategies.

$Str_{x,y}$ are formed by *JSM* operators $\bar{O}_{x,y}(\Omega(p))$ and functions of abductive hypothesis acceptance ρ^+(p) and $\rho^-(p)$. $\bar{O}_{x,y}(\Omega(p))$ carry out sequential application of p.i.r.-1 (σ) (inductive inference rules) and p.i.r.-2 (inference rules by analogy), which are $(I)^\sigma_{x,y}$ and $(II)^\sigma_{x,y}$, where $\sigma = +, -, 0, \tau$,

$$(I)_{x,y} = \{(I)^+_{x,y}, (I)^-_{x,y}, (I)^0_{x,y}, (I)^\tau_{x,y}\},$$

$$(II)_{x,y} = \{(II)^+_{x,y}, (II)^-_{x,y}, (II)^0_{x,y}, (II)^\tau_{x,y}\},$$

where $(I)_{x,y}(\Omega(p)) = \tilde{\Delta}_{x,y}(p), \bar{O}_{x,y}(\Omega(p)) = \tilde{\Omega}_{x,y}(p)$.

We note that p.i.r.-2 uses the results of applying p.i.r.-1, and $\bar{O}_{x,y}(\Omega(p))$, $\rho^+(p)$, $\rho^-(p)$, ..., $\bar{O}_{x,y}(\Omega(s))$ completes the *JSM* reasoning.

In $\bar{O}_{x,y}(\Omega(s))$, $\rho^+(s)$, $\rho^-(s)$ the procedure of abductive hypothesis acceptance by $\rho^\sigma(s)$ is realized if $\rho^\sigma(s) \ge \bar{\rho}^\sigma$, where $\bar{\rho}^\sigma$ is the given threshold.

The heuristic of the ASSR JSM method formalizes the minimization of randomness in the expansion of the *FB*(p) by using the histories of possible worlds HPW $\Omega(p, h)$, $h = 1, \ldots, (s + 1)!$

The **Basis** of QAT $\tilde{\mathfrak{I}}_{x,y}(p)$ for $Str_{x,y}$ and FB(p) will be called $\mathfrak{I}_{x,y}(p) = \langle \Sigma, \Omega(p, h), \mathfrak{R}\rangle$, where Σ is an open set of axioms containing descriptive axioms, axioms of data structures, procedural axioms representing p.i.r.-1 and p.i.r.-2 declaratively, axioms characterizing JSM reasoning (for example, CCA$^\sigma$ or ($\exists^\sigma$), where $\sigma = +, -$) and updated empirical regularities, that is, realizations of JCF. $\mathfrak{R}$ is a set of plausible inference rules (p.i.r.-1, p.i.r.-2) and the rules of deductive inference.

QAT is defined by the JSM closure of the basis $[\mathfrak{I}_{x,y}(p, h)]$, where $[\mathfrak{I}_{x,y}(p, h)] = \langle \Sigma, \tilde{\Omega}_{x,y}(p, h) \cup \tilde{\Delta}_{x,y}(p, h), \mathfrak{R}\rangle$, $\bar{O}_{x,y}(\Omega(p, h)) = \tilde{\Omega}_{x,y}(p, h)$, $(I)^\sigma_{x,y}(\Omega(p, h)) = \tilde{\Delta}_{x,y}(p, h)$ for $p = 0, 1, \ldots, s$, values of h are HPW_r, where $r = 1, \ldots, (s + 1)!$.

The histories of possible worlds HPW_r will be represented by their numbers $r = 1, \ldots, (s + 1)!$.

$[\mathfrak{I}_{x,y}(p, h)]$ is determined by applying the JSM reasoning to $\Omega(p, h)$ before stabilization, when the application of p.i.r.-1 does not generate new hypotheses, and $\rho^\sigma(s) \ge \bar{\rho}^\sigma$, where $\sigma = +, -$.

We consider $\tilde{\mathfrak{I}}_{x,y}(p, h) = [\mathfrak{I}_{x,y}(p, h)]$ for $0 \le p \le s$ and all h from $\overline{HPW}$:

$$\mathfrak{I}_{x,y} = \{\tilde{\mathfrak{I}}_{x,y}(p, h) \mid (1 \le p \le s) \,\&\, (h \in \overline{HPW})\}.$$

$\mathfrak{I}_{x,y}$ generates $\mathrm{G}_{x,y} = \left(\bigcup_{\chi\in E^+}\{\langle V, Y\rangle \mid A^+_\chi(V, Y)\}\right) \cup$

Table 5. Reasoning quality assessment scale

(*)	DR	$Str_{x,y}$	EL	ET	WET	$\dfrac{l_0}{m_0}$	a	b	c	ρ^+	ρ^-

$\left(\bigcup_{\chi\in E^-}\{\langle V,Y\rangle \mid A_\chi^-(V,Y)\}\right)$, where $A_\chi^+(V,Y)$ are realizations of ICF for values $\langle C',Q\rangle^+$ of pairs $\langle V, Y\rangle$.

Then $\langle C',Q\rangle^\sigma \in$ G, where σ = +, –. Therefore, $\exists n(h)\, J_{\langle \nu, n(h)\rangle} H_2(C',Q,\overline{s},h)$, where v = 1, –1, giving rise to $A_\chi^\sigma(C',Q)$ – realization of ICF from $\overline{ICF}$.

Thus, the set $G_{x,y}$ for $Str_{x,y}$ has one-to-one correspondence with Σ_E, that is, the set $\overline{RICF}$ of all realizations of $\overline{ICF}$.

Then, we define E as the closure $\Im_{x,y}$.

***Df.*19-4. E-closure** of the set $\Im_{x,y}$ will be called $[\Im_{x,y}]_E = \langle \Sigma \cup \Sigma_E, \tilde{\Omega}_{x,y}(\overline{s},(\overline{s}+1)!) \cup \tilde{\Delta}_{x,y}(\overline{s},(\overline{s}+1)!), \Re\rangle$, where $\rho^\sigma(\overline{s},(\overline{s}+1)!) \geq \overline{\rho}^\sigma$, σ = +, –, Σ_E is the set of all $A_\chi^\sigma(C',Q)$, corresponding to $G_{x,y}$, where $\chi \in E$.

Obviously, the set of all E-closures $\Im_{x,y}$ for $Str_{x,y} \in \overline{Str}$ $\Im = \{[\Im_{x,y}]_E\} \mid (x \in I^+) \mathbin{\underline{\&}} (y \in I^-)\}$ is an extensional *ExtER* generated by $\overline{\overline{IntER}}$.

Thus, for the strategy of JSM reasoning $Str_{x,y}$, we obtain a scheme of JSM research:

1^0. $\Omega_0(0, 1), \Omega(1, 1), \ldots, \Omega(s, 1)$;

$\Omega(0, 1) \subset \Omega(1, 1) \subset \ldots \subset \Omega(s, 1)$;

2^0. $\Omega^\tau(0, 1)$, $\Omega^\tau(0, 1) = \Omega^\tau(p, h)$ for all p and h, where $0 \leq p \leq s$, $h \in \overline{HPW}$;

3^0. $\overline{HPW}$, $\left|\overline{HPW}\right| = (s+1)!$;

4^0. $Str_{x,y}$

5^0. $\overline{ICF}$

6^0. $[\Im_{x,y}]_E$

$\Im_E = \{[\Im_{x,y}]_E \mid (x \in I^+) \,\&\, (y \in I^-\}$ represents the result of JSM research containing a set of empirical nomological statements of the form $A_\chi^\sigma(C',Q)$, where $\chi \in E, \sigma = +, -$, to which *ExtER* corresponds, represented in the Scheme *Concept ER* through branches $B_{r_r}(\chi)$, where $1 \leq r \leq \rho$.

In [6], two scales **for assessing the quality of reasoning and hypotheses** were formulated necessary for accepting the results of a *JSM* research. Through m_0 and l_0, $|\Omega^\tau(0)|$ and the set and number of correct predictions of the studied effect were denoted. $m_0 = l_0 + a + b + c$, where a, b, c is the type and number of prediction errors such that "a" is the type of erroneous predictions "1" instead of "–1 " or "–1 " instead of "1," "b" is the type of erroneous predictions "0" (actual contradiction) instead of "1" or "–1 "; and "c" is the type of erroneous predictions τ (uncertainties) instead of "1" or "–1 " (that is, rejection of predictions). By $\dfrac{l_0}{m_0}$ in [6], the degree of reliability of predictions in the JSM research was indicated. In Tables 5 and 6, we give the mentioned scales.

Scales (*) and (**) are a means of **accepting** the results of JSM research. They can be expanded and enriched by the Scheme *Concept ER*, Definitions *Df.*18-4, *Df.*19-4 and further formulated *Df.*20-4.

***Df.*20-4.** JSM research will be called **consistent** if there is a strategy of JSM reasoning $Str_{x,y}$ such that the E-closure $\Im_{x,y}$ is nonempty: $\neg([\Im_{x,y}]_E = \varnothing)$; a JSM research will be called **acceptable** if there is a strategy $Str_{x,y}$ such that $\neg([\Im_{x,y}]_E = \varnothing)$ and $l_0 > m_0 - (a + b + c)$; a JSM research will be called **fruitful** if, for each of the m_0 elements of $\Omega^\tau(0)$ there exists a strategy $Str_{x,y}$ such that its results belong to $[\Im_{x,y}]_E$ and $l_0 = m_0$.

The type of forest obtained for *ExtER* and *Df.*20-4 is informatively characterized by JSM research, enriching the scales (*) and (**). It seems useful to compile and compare the results of JSM research using enriched scales (*) and (**) for both various domains and for ongoing JSM research presented in the generated open QAT.

In [1, 2], an abduction inference was determined that performs abduction of the second kind for each of the modalities M_χ (M_χ is $\Box_\chi$, $\Diamond_\chi$, ∇_χ), where $\chi \in E = \{a, b, \ldots, m, n\}$.

We consider the case for $\Box_a \Box_a A_2^+(C',Q)$:

$$\Box_a \forall Z((\hat{L}_2^+(C',Q,\overline{s},\overline{h}) \,\&\, P(Z,\overline{s},\overline{h}) \,\&\, (C' \subset Z)) \rightarrow \hat{L}_1^+(Z,Q,\overline{s},\overline{h}))$$

$$\forall Z((C' \subset Z) \rightarrow Ver[\hat{L}_1^+(Z,Q,\overline{s},\overline{h})] = t)$$

$$\overline{\Box_a \hat{L}_2^+(C',Q,\overline{s},\overline{h})}$$

Table 6. Hypothesis quality assessment scale

(**)	DR	$Str_{x,y}$	$\bar{v}$	k	EL	ET	WET	$H_2(V, Y, p, h)$	$H_1(Z, Y, p, h)$

EXPLANATORY NOTE: $\bar{v} = \langle v, n\rangle$, where $v \in \{1, -1, 0\}$; k is the number of examples that generated a hypothesis, H_2, H_1 are the predicates for hypotheses about causes and hypotheses about predictions, respectively; ρ^+ and ρ^- are the functions of the degree of abductive acceptance of hypotheses; and RD denotes two possible variants for applying plausible inference rules (p.i.r.-1 and p.i.r.-2) using the isomorphism p.i.r.-$1^{(\sigma)}$, where $\sigma = +, -, 0, \tau$, or without it [6].

$\mathrm{Ver}[\hat{L}_1^+(Z, Q, \bar{s}, \bar{h})]$ is the function of **verification** of predictions, which performs **correspondent** truth, establishing compliance with the available fact; and $V[A_2^+(C', Q] = t$ in MJL by virtue of Proposal 1-2.

Similar rules of inductive inference are formulated for other modalities of M_χ.

Thus, the modal operators M_χ, where $\chi \in E$, from implicative statements, where $1 \le j \le 6$, are ported to the antecedent provided that the consequent of the implication in A_j^σ is verified. The proposed formalization of abduction of the second kind in the ASSR JSM method is a **refinement** of the idea of C.S. Pierce about abduction from his famous text [15][13], as the ampliative inference **rules** [37].

We note that the causal completeness axioms $\mathrm{CCA}^{(\sigma)}$, where $\sigma = +, -$, and the functions $\rho^\sigma(p)$ are a refinement of the idea of C.S. Pierce about abduction as a means of accepting the generated hypotheses [16]. Earlier, using $\mathrm{CCA}^{(\sigma)}$ and $\rho^\sigma(\mathrm{p})$, abduction of the first kind was determined, which, as a component of the JSM reasoning, is used to apply abduction of the second kind. The interaction of the two abductions consists in the fact that **semi-hypotheses** about causes and semi-hypotheses about predictions are accepted through abduction of the first kind, and **hypotheses** about reasons and hypotheses about predictions with modal operators M_χ that express the modal degree of validity of hypotheses by partial order relations $\sqsupseteq$ on the set $E = \{a, b, \ldots, m, n\}$, where $\chi \in E$, are generated through abduction of the second kind.

The status of hypotheses is based on minimizing the randomness of extensions in the histories of possible worlds for all their sets $\overline{HPW}$. Thus, the problem of determining physical (nonlogical) modalities is solved using **empirical nomological statements** that are the result of JSM research. The problem of determining nomological statements using logic, as already noted, was systematically developed by Hans Reichenbach in [30, 31, 38, 39]. We should also mention the remark of *R.* Feys [40] on the connection of the idea of causality and modalities.

In §5 we will begin the study of the family of modal logics ERA generated by JSM research, the result of which are empirical regularities and their adoption through abduction of the second kind. The semantic foundations of the logic of the ERA family are **finite** sets of histories of **finite** possible worlds. The source of the appearance of the ERA family logics is the classification of the intensional $\mathrm{Int}\overline{\overline{ER}}$, represented by the Tree T expressing a set of integral causal forcings $\overline{ICF}$, by which realizations ICF are defined, that is, initial CF and their descendants. Initial CF and their descendants define modalities of M_χ, where $\chi \in E$, and $\bar{E}\ \langle E, \sqsupseteq\rangle$ is a partially ordered set with the largest element a and the smallest element n.

[13]An interpretation of [15] is available in [18], where the condition of the best explanation is added.

5. *ERA* MODAL LOGICS GENERATED BY JSM RESEARCH

JSM research is formed by applying the JSM reasoning to the sequence of expanded fact bases FB(p, h), where $p = 0, 1, \ldots, s$, such that they correspond to the sequence of representations of FB(p, h), where $h = 1, \ldots, (s + 1)!$, using elementary formulas $J_{\bar{v}}H_1$ (Z, Y, p, h), where $\bar{v} = \langle v, 0\rangle$, $v = 1, -1$, or $v = \tau$ and $\bar{v} = (\tau, 0)$, and h is a variable for histories of possible worlds HPW_h from the generated set of histories of possible worlds corresponding to initial HPW_1.

Modified operators $\Box$ (necessity), $\Diamond$ (possibility), ∇ (weak possibility) are defined by integral causal forcing ICF from the set $\overline{ICF}$, corresponding to the set of all the histories of possible worlds $\overline{HPW}$. We note that this means that the semantics of a **finite** set of histories of **finite** possible worlds constructively generated are given. Modal operators $\Box_{\chi_1}, \Diamond_{\chi_2}, \nabla_{\chi_3}$ were defined by assessments functions $\tilde{V}[M_\chi\varphi]$, where $\chi_1 = \{a, b, c, d, e, f, h\}$, $\chi_2 = \{i, j, k, l\}$, $\chi_3 = \{m, n\}$.

As for classification of $\overline{ICF}$ forming the intensional of the concept of empirical regularities, it is represented by the Tree T. However, the modal logics of the ERA family are "empirical regularities completed by abduction of the second kind," considered below, will correspond to the **simplifications** of the Tree T, which are Trees T_1 and T_2 from §3. The question of the possibility of formalizing multimodal logics corresponding to the Tree T, i.e., to its fourteen branches Br(χ), is open. In this regard, we consider a simpler modal Propositional logic ERA_0 corresponding to the Tree T_1.

Trees T_1 and T_2, presented in §3, characterize a partially ordered set of modalities, which one-to-one corresponds to the set $E = \{a, b, ..., m, n\}$. T_1 and T_2 generate sets $\{\square, \diamond\}$ and $\{\square, \diamond, \nabla\}$, respectively.

The monotony condition $\rho^{\sigma}(p)$, taken into account in Tree T is omitted; histories of possible worlds $HPW_h = \{\Omega(0, h), \Omega(1, h), ..., \Omega(i, h), ..., \Omega(s, h)\}$, where $0 \le i \le s$ and $1 \le h \le (s + 1)!$, are considered.

Tree vertices T_1 $\langle v, v\rangle$, $\langle\tau, v\rangle$ correspond to codes $Cd^{(1)} = v...v \bullet v...v$ and $Cd^{(2)} = \underbrace{\tau...\tau}_{q} v...v \bullet \underbrace{\tau...\tau}_{q} v...v$, for branches Br_1 and Br_2, where $1 \le q < s + 1$. The vertices of the Tree T_2 correspond to codes $Cd^{(1)} = v...v \bullet v...v$, $Cd^{(2)} = \underbrace{\tau...\tau}_{q}\underbrace{v...v}_{s+1-q} \bullet \underbrace{\tau...\tau}_{q}\underbrace{v...v}_{s+1-q}$, where $2q < s + 1$; $Cd^{(3)} = \underbrace{\tau...\tau}_{q}\underbrace{v...v}_{s+1-q} \bullet \underbrace{\tau...\tau}_{q}\underbrace{v...v}_{s+1-q}$, where $2q \ge s + 1$, for branches Br_1, Br_2 and Br_3.

We define an estimation function $\tilde{V}[M_\chi \varphi]$ for ERA_0 formulas with modal operators M_χ, corresponding to the Tree T_1 using ICF from the set $\overline{ICF}$.

Signature ERA_0: Propositional variables **p**, **q**, **r**,..., (which may have lower indices), logical connectives: $\neg, \&, \vee, \rightarrow, \square, \diamond$; and auxiliary characters, – (,).

The definition of formula ERA_0 is standard [40, 41].

Axioms ERA_0

($\square$2) $\square\, \mathbf{p} \rightarrow \mathbf{p}$

($\diamond$2) $\diamond\mathbf{p} \rightarrow \mathbf{p}$

($\square$3) $\square\, (\mathbf{p} \,\&\, \mathbf{q}) \leftrightarrow (\square\mathbf{p} \,\&\, \square\mathbf{q})$

($\square$4) $\square(\mathbf{p} \vee \mathbf{q}) \leftrightarrow (\square\mathbf{p} \vee \square\mathbf{q})$

($\diamond$3) $\diamond(\mathbf{p} \,\&\, \mathbf{q}) \leftrightarrow (\diamond\mathbf{p} \,\&\, \diamond\mathbf{q})$

($\diamond$4) $\diamond(\mathbf{p} \vee \mathbf{q}) \leftrightarrow (\diamond\mathbf{p} \vee \diamond\mathbf{q})$

($\neg\square$) $\neg\square\mathbf{p} \rightarrow (\diamond\mathbf{p} \vee \neg\mathbf{p})$

($\neg\diamond$) $\neg\diamond\mathbf{p} \rightarrow (\square\mathbf{p} \vee \neg\mathbf{p})$

($\square\square$) $\square\square\, \mathbf{p} \rightarrow \square\mathbf{p}$

($\square\diamond$) $\square\diamond\mathbf{p} \rightarrow \diamond\mathbf{p}$

($\diamond\square$) $\diamond\square\mathbf{p} \rightarrow \neg\mathbf{p}$

($\diamond\diamond$) $\diamond\diamond\, \mathbf{p} \rightarrow \neg\mathbf{p}$

($\square\&\diamond$) $\neg(\square\mathbf{p} \,\&\, \diamond\mathbf{p})$

($\square\,\&\neg$) $\neg(\square\mathbf{p} \,\&\, \neg\mathbf{p})$

($\diamond\&\neg$) $\neg(\diamond\mathbf{p} \,\&\, \neg\mathbf{p})$

($\square\neg$) $\neg\square\neg\mathbf{p}$

($\diamond\neg$) $\neg\diamond\neg\mathbf{p}$

($\square\neg\diamond$) $(\square\mathbf{p} \rightarrow \neg\diamond\mathbf{p})$

$\varphi \leftrightarrow \psi \rightleftharpoons (\varphi \rightarrow \psi) \,\&\, (\psi \rightarrow \varphi)$,

$f \rightleftharpoons \mathbf{p} \,\&\, \neg\, \mathbf{p}$

$t \rightleftharpoons \neg f$

The two-valued Propositional logic L_2 is set.

By "$\vdash$" we denote the provability of the formulas ($\vdash \varphi$) and the derivability ($\varphi_1, ..., \varphi_n \vdash \psi$).

Inference rules:

R1. $\varphi, \varphi \rightarrow \psi \vdash \psi$;

R2. $\varphi(\mathbf{p}) \vdash \varphi(\chi)$, $\varphi(\chi) = \int_{\mathbf{p}}^{\chi} \varphi(\mathbf{p})\mid$ are substitution rules;

R3. $\square\varphi, \square(\varphi \rightarrow \psi) \vdash \square\psi$;

R4. $\diamond\varphi, \diamond(\varphi \rightarrow \psi) \vdash \diamond\psi$;

R5. $\varphi(\chi) \vdash \varphi(\chi_1)$, where $\chi \leftrightarrow \chi_1$ is the rule of replacement of equivalent formulas.

Proposal 4-5 holds. ERA_0 is controversial.

From ($\neg\square$) we obtain $\square\mathbf{p} \vee \diamond\mathbf{p} \vee \neg\mathbf{p}$; applying R2 to $\square\mathbf{p} \vee \diamond\mathbf{p} \vee \neg\mathbf{p}$ we obtain $\square\neg\mathbf{p} \vee \diamond\neg\mathbf{p} \vee \neg\neg\mathbf{p}$, $\square\neg\mathbf{p} \vee \diamond\neg\mathbf{p} \vee \mathbf{p}$, but $\square\neg\mathbf{p} \leftrightarrow f$, $\diamond\neg p \leftrightarrow f$ by virtue of ($\square\neg$) and ($\diamond\neg$).

Therefore, applying R5, we obtain $f \vee f \vee \mathbf{p}$.

Therefore, $\vdash\mathbf{p}$, and therefore any formula φ is provable: $\vdash\varphi$ – ERA_0 is an absolutely contradictory calculus.

We formulate the $ERA_{0,1}$ calculus by restricting the substitution rule R2 in the ERA_0 calculus: R*2 is $\varphi(p) \vdash \varphi(\chi) = \int_{\mathbf{p}}^{\chi} \varphi(\mathbf{p})\mid$, where the formulas χ correspond to **monotone** Boolean functions from the set M^*, where M^* is the set of all superpositions of the set $\{p \,\&\, q, p \vee q\}$: $M^* = [\{p \,\&\, q, p \vee q\}]$ is the closing $\{p \,\&\, q, p \vee q\}$.

The formulas φ, which are valid in the ERA logics, will be denoted by $\models\varphi$.

Thus, $ERA_{0.1}$ is formed by ERA_0 axioms and inference rules R1,R*2, R4 and R5.

The logics of the ERA family arise on the basis of *JSM* research formed by applying JSM reasoning to the set of histories of possible worlds $\overline{HPW}$. Each HPW_h is a sequence of $\Omega(0, h)$, $\Omega(1, h)$, ..., $\Omega(s, h)$ such that $\Omega(0, h) \subset ... \subset \Omega(s, h)$, and $h = 1, ..., (s + 1)!$.

HPW_h correspond to three types of codes of empirical laws $v...v$ and $\tau...\tau v...v$, which are codes of empirical laws and empirical tendencies, respectively, that is, they form regular codes ($v = 1, -1$); sequences $\theta_1...\theta_s$, where $\theta_i = 0, -1, 1, \tau$, other than $v...v$ and $\tau...\tau v...v$, are irregular and represent the **absence of empirical regularities**, $\overline{HPW}$ in general form can be represented by (*) so that the terms of the partition can be empty.

(*) $\overline{HPW} = \overline{HPW}_v \cup \overline{HPW}_\tau \cup \overline{IHPW}$, where $\overline{HPW}_v$, $\overline{HPW}_\tau$ and $\overline{IHPW}$ are sets of histories corresponding to empirical laws, empirical tendencies and their absence (they have irregular codes).

The specific values of the Propositional variables of the ERA family logics are sequences $H_2(C', Q, \overline{p}_0, \overline{s}, \overline{h})$, $H_2(C', Q, \overline{p}_1, \overline{s}, \overline{h}), ..., H_2(C', Q, \overline{p}_s, \overline{s}, \overline{h})$ and their corresponding sequences $H_1(Z, Q, \overline{p}_0, \overline{s}, \overline{h})$, $H_1(Z, Q, \overline{p}_1, \overline{s}, \overline{h}), ..., H_1(Z, Q, \overline{p}_s, \overline{s}, \overline{h})$, for all Z that represent causal forcing A_i^{σ}, where $1 \le j \le 6$, for all Z that represent causal forcing $L_1^{\sigma}(Z, Q, p, \overline{s}, \overline{h})$ from $L_2^{\sigma}(C', Q, p, \overline{s}, \overline{h})$ for all p and Z, expressed by implication $\rightarrow$, which is representable by codes Cd.

Thus, the values of Propositional variables are sequences represented by statements that express the realizations of predicates $L_2^{\sigma}(V,Y,p,s,h)$ and $L_1^{\sigma}(Z,Y,p,s,h)$. These sequences, in turn, correspond to **regular** codes of the type *v...v* and τ...τ*v...v*; and also irregular codes $Cd\ \theta_1...\theta_s$ different from them such that $\theta_i \in \{1, -1, 0, \tau\}$, where $1 \le i \le s$. Consequently, a set of all histories of possible worlds can be represented by $(*)\overline{HPW} = \overline{HPW}_{\mathrm{v}} \cup \overline{HPW}_{\tau} \cup \overline{IHPW}$, where $\overline{HPW}_{\mathrm{v}}$, $\overline{HPW}_{\tau}$ and $\overline{IHPW}$ correspond to *Cd* of types *v...v*, τ...τ*v...v* and irregular codes $\theta_1...\theta_s$.

In accordance with the Tree T_1 of the classification of empirical regularities, we obtain that the following **assessment functions** for □**p**, ◇**p** and ⌉**p** and the variable *h* with a range of $\overline{HPW}$ are definable: (1) $V[\Box\mathbf{p}] = t$, if and only if for all *h* **p** takes the value $\underbrace{v...v}_{s}$, i.e. $Cd(p, h) = v...v$ for all HPW; (2) $V[\Diamond p] = t$ if and only if **p** takes the value $\underbrace{\tau...\tau v...v}_{s}$; i.e., there exists h such that $Cd(p, h) = \underbrace{\tau...\tau v...v}_{s+1}$ and for all *h*, *Cd*(*p*, *h*) are regular codes, where *Cd*(*p*, *h*) is the value of the variable *h* corresponding to HPW_h. (3) $V[\rceil p] = t$ if and only if there exists *h* such that HPW_h, such that $Cd(p, h) = \theta_1...\theta_{s+1}$, where $\theta_1...\theta_{s+1}$ is an irregular code, i.e., the code *Cd* such that it is not *v...v* or τ...τ*v...v*, where $v = 1, -1$, and $\theta_i \in \{1, -1, 0, \tau\}$. Therefore, we can define a function *G*(*p*, *h*) such that *G* displays $\Im \times \overline{HPW}$ in $\{\Box\mathbf{p}, \Diamond\mathbf{p}, \rceil\mathbf{p}\}$, where $\mathbf{p} \in \Im$, $HPW_h \in \overline{HPW}$, $h = 1, ..., (s+1)!$, i.e, $G : \Im \times \overline{HPW} \to \{\Box\mathbf{p}, \Diamond\mathbf{p}, \rceil\mathbf{p}\}$.

In this way,

$$\mathrm{G}(\mathbf{p},h) = \begin{cases} \Box\mathbf{p}, \text{ if } Cd(p,h) \in \overline{HPW_{\mathrm{v}}} \text{ for all } h; \\ \Diamond\mathbf{p}, \text{ if there exists } h_1 \text{ such that} \\ Cd(p,h_1) \in \overline{HPW}_{\tau} \text{ and for all } h \text{ such that} \\ \neg(h = h_1)\, h \in \overline{HPW}_{\mathrm{v}} \cup \overline{HPW}_{\tau}; \\ \rceil\mathbf{p} \text{ if there exists h such that} \\ Cd(\mathbf{p},h) \in \overline{IHPW}. \end{cases}$$

Obviously, the function G(**p**, *h*) represents the Tree T_1. We introduce the metacharacter |= and determine the **truth of the formulas φ** of the $ERA_{0,1}$ logic in the histories of possible worlds HPW_h, where $HPW_h \in \overline{HPW}$, and $\overline{HPW} = \overline{HPW}_{\mathrm{v}} \cup \overline{HPW}_{\tau} \cup \overline{IHPW}$, $\left|\overline{HPW}\right| = (s+1)!$.

1^0. $HPW_h \models \mathbf{p}$, if and only if $Cd(\mathbf{p}, h) = v...v$ or $Cd(\mathbf{p}, h) = \tau...\tau\ v...v$, where $v = 1, -1$;

-2^0. it is not true that $HPW_h \models f$;

3^0. $HPW_h \models \neg\mathbf{p}$, if and only if it is not true that $Cd(\mathbf{p}, h) = v...v$ and it is not true that $Cd(\mathbf{p}, h) = \tau...\tau v...v$; i.e. $\mathbf{Cd(p, h)} \in \overline{IHPW}$.

3^0. $HPW_h \models (\varphi \,\&\, \psi)$, if and only if $HPW_h \models \varphi$ and $HPW_h \models \psi$;

4^0. $HPW_h \models (\varphi \vee \psi)$, if and only if $HPW_h \models \varphi$ or $HPW_h \models \psi$;

5^0. $HPW_h \models (\varphi \to \psi)$, if and only if $HPW_h \models \varphi$, then $HPW_h \models \varphi$;

6^0. $HPW_h \models (\varphi \leftrightarrow \psi)$, if and only if $HPW_h \models \varphi$, if and only if $HPW_h \models \psi$;

7^0. $HPW_h \models \Diamond\varphi$, if and only if $Cd(\mathbf{p}, h) = v...v$ for all *h*;

8^0. $HPW_h \models \Diamond\varphi$, if and only if there exist h_1 such that $Cd(\mathbf{p}, h_1) = \tau...\tau v...v$ and for all such *h* that $Cd(\mathbf{p}, h) = \tau...\tau v...v$ or $Cd(\mathbf{p}, h) = v...v$, where $\overline{HPW}_h \in \overline{HPW}_{\mathrm{v}} \cup \overline{HPW}_{\tau}$;

9^0. $HPW_h \models \Diamond(\varphi \vee \psi)$, if and only if $HPW_h \models \Diamond\varphi$ or $HPW_h \models \Diamond\psi$ for all *h* from $\overline{HPW}$;

10^0. $HPW_h \models \Box(\varphi \vee \psi)$, if and only if $HPW_h \models \Box\varphi$ or $HPW_h \models \Box\psi$ for all *h* from $\overline{HPW}$;

11^0. $HPW_h \models \Box(\varphi \,\&\, \psi)$, if and only if $HPW_h \models \Box\varphi$ and $HPW_h \models \Box\psi$ for all *h* from $\overline{HPW}$;

12^0. $\mathrm{HPW_h} \models \Diamond(\varphi \,\&\, \psi)$, if and only if $HPW_h \models \Diamond\varphi$ and $HPW_h \models \Diamond\psi$ for all *h* from $\overline{HPW}$.

In case $V[\Box\mathbf{p}] = t$ $\overline{HPW} = \overline{HPW}_{\mathrm{v}}$, and $\overline{HPW} = \varnothing$ and $\overline{IHPW} = \varnothing$. In case $V[\Diamond\mathbf{p}] = t$ $\overline{HPW} = \overline{HPW}_{\mathrm{v}} \cup \overline{HPW}_{\tau}$, $\overline{HPW}_{\tau} \ne \varnothing$ and $\overline{IHPW} = \varnothing$. If $V[\rceil p] = t$, then $\overline{IHPW} \subseteq \overline{HPW}$ and $\overline{IHPW} \ne \varnothing$. Hence, $\overline{IHPW} \ne \varnothing$ is a necessary and sufficient condition for the absence of empirical regularities.

<u>Remark 14-5.</u> We formulate an assumption regarding the interpretation of the iteration of the modalities □ and ◇. We consider the possible cases ◇□**p**, □◇**p**, □□**p** and ◇◇**p**. We take the direction of adding a modal operator from the variable *p* to the left side, which will be represented by the extension of the corresponding codes *Cd*(**p**, *h*):

$$(1)\ \Diamond\Box\text{:}\ \underbrace{v...v}_{\Box}\ \underbrace{\tau...\tau v...v}_{\Diamond} - \rceil\mathbf{p},$$

$$(2)\ \Box\Diamond\text{:}\ \underbrace{\tau...\tau v...v}_{\Diamond}\underbrace{v...v}_{\Box} - \Diamond\mathbf{p},$$

$$(3)\ \Box\Box\text{:}\ \underbrace{v...v}_{\Box}\underbrace{v...v}_{\Box} - \Box\mathbf{p}.$$

$$(3)\ \Diamond\Diamond\text{:}\ \underbrace{\tau...\tau v...v}_{\Diamond}\underbrace{\tau...\tau v...v}_{\Diamond} - \rceil\mathbf{p}.$$

In (1) we have an irregular resulting *Cd*, and therefore the formula ◇□ → ⌉**p** is valid (axiom);

In (◇□) according to the definition of $HPW_h \models \urcorner$p, since $v...v\tau...\tau v...v$ is an irregular *Cd*.

In (2) we have a regular resultant *Cd* $\tau...\tau v...vv...v$, and therefore the formula $\Box\Diamond\mathbf{p} \to \Diamond\mathbf{p}$ is valid.

In (3) we have a regular resulting *Cd* $v...vv...v$, and therefore the formula $\Box\Box\mathbf{p} \to \Box\mathbf{p}$ is valid.

In (4) we have an irregular resulting *Cd* $\tau...\tau v...v\tau...\tau v...v$, and therefore, according to the definition of $HPW_h \models \urcorner\mathbf{p}$, the formula $\Diamond\Diamond\mathbf{p} \to \urcorner\mathbf{p}$ is valid.

Lemma 1-5. Axioms of $ERA_{0.1}$ are valid.

We have found that (◇□)$\Diamond\Box\mathbf{p} \to \urcorner\mathbf{p}$ and (◇◇)$\Diamond\Diamond\mathbf{p} \to \urcorner\mathbf{p}$ are valid with respect to $\overline{HPW}$.

(⌉□) $\urcorner\Box\mathbf{p} \to (\Diamond\mathbf{p} \vee \urcorner\mathbf{p})$ and (⌉◇) $\urcorner\Diamond\mathbf{p} \to (\Box\mathbf{p} \vee \urcorner\mathbf{p})$ are valid, since $\Box\mathbf{p} \vee \Diamond\mathbf{p} \vee \urcorner\mathbf{p}$ is valid, that is, the law of the excluded fourth, which follows from the definition of the function $G(\mathbf{p}, h)$.

(□2) $\Box\mathbf{p} \to \mathbf{p}$ is valid by virtue of the definition $HPW_h \models \Box\mathbf{p}$ for all h: $Cd(p, h) = v...v$.

We note that for the sake of simplicity of recording $Cd(p, h)$ $v...v \bullet v...v$ and $\tau...\tau v...v \bullet \tau...\tau v...v$, *that is,* for regular *Cd* and $\theta_1...\theta_{s+1} \cdot \mu_1...\mu_{s+1}$ while for irregular *Cd* we will represent with the types $v...v$, $\tau...\tau v...v$, $\theta_1...\theta_{s+1}$ corresponding to them.

(□3) $\Box(p \,\&\, q) \leftrightarrow (\Box p \,\&\, \Box q)$ is valid, since $Cd(p \,\&\, q) = v...v$ is equivalent to $Cd(p) = v...v$ and $Cd(q) = v...v$.

(□4) $\Box(p \vee q) \leftrightarrow (\Box p \vee \Box q)$ is valid, since $Cd(p \vee q) = v...v$ is equivalent to $Cd(p) = v...v$ or $Cd(q) = v...v$.

(◇2) $\Diamond(p \,\&\, q) \leftrightarrow (\Diamond p \,\&\, \Diamond q)$ is valid since $Cd(p \,\&\, q) = \tau...\tau v...v$, and $(p \,\&\, q) \leftrightarrow (q \,\&\, p)$, therefore, if $Cd(p) = \tau...\tau v...v$ and $Cd(q) = v...v$, we obtain $Cd(p \,\&\, q) = \tau...\tau v...vv...v$, but $Cd(p \,\&\, q) = v...v\tau...\tau v...v$, *that is,* irregular *Cd*. Therefore $Cd(p) = Cd(q) = \tau...\tau v...v$ for all h such that $\overline{HPW} \in \overline{HPW_v} \cup \overline{HPW_\tau}$.

(◇3) $\Diamond(p \vee q) \leftrightarrow (\Diamond p \vee \Diamond q)$ is valid by virtue of validity of $p \to (\Box p \vee \Diamond p)$ and condition 9^0 from the definition of the truth of the formulas φ of the $ERA_{0,1}$ logic: $HPW_h \models \Diamond(\varphi \vee \psi)$ if and only if $HPW_h \models \Diamond\varphi$ or $HPW_h \models \Diamond\psi$ for all h from $\overline{HPW}$.

(□ & ⌉) $(\Box p \,\&\, \urcorner p) \to f$ is valid since $V[\Box p] = t$ if and only if $Cd(p, h) = v...v$ for all h and $V[\urcorner\mathrm{p}] = \mathrm{t}$ if and only if there exists h such that $Cd(p, h) = \theta_1...\theta_{s+1}$, where $\theta_1...\theta_{s+1}$ is an irregular code corresponding to $\overline{IHPW}(\overline{IHPW} \cap \overline{HPW_v} = \varnothing)$.

Similarly, the validity of (◇ & ⌉) $(\Diamond p \,\&\, \urcorner p) \leftrightarrow f$ is established. (□⌉) $\Box\urcorner p \to f$ is valid since $V[\Box\varphi] = t$, if and only if for all h $HPW_h \models \varphi$, $HPW_h \in \overline{HPW}$; and $V[\urcorner p] = t$, if and only if $Cd(p, h)$ is an irregular code and there exists h such that $Cd(p, h)$ corresponds to an element from $\overline{IHPW}(\overline{IHPW} \cap \overline{HPW_v} = \varnothing)$.

Similarly, the validity of (◇ & ⌉) $\Diamond\urcorner p \to f$ is established. (◇) $\Diamond p \to p$ is valid since $V[\Diamond p] = t$ if $HPW_h \models p$ for all h such that $Cd(p, h) = \tau...\tau v...v$.

(□□) $\Box\Box p \to \Box p$ is valid, since $Cd(\Box\Box p) = v...vv...v$ implies $Cd(\Box p) = v...v$.

(□◇) $\Box\Diamond p \to \Diamond p$ is valid, since $Cd(\Box\Diamond p) = \tau...\tau v...vv...v$ implies $Cd(\Diamond p) = \tau...\tau v...v$.

Obviously, (□¬◇) is valid.

Lemma 1-5 is proved: all axioms of $ERA_{0.1}$ are valid.

There is also a lemma on the correctness of the $ERA_{0.1}$ inference rules.

Lemma 2-5. The inference rules R1, R*2, R3, R4, and R5 of the $ERA_{0.1}$logic remain valid: if the parcels of the rules are valid, then their consequences are valid.

Formula φ is valid in $ERA_{0.1}$ if and only if $V[\varphi] = t$ for all assessments *V*.

For valid formulas, we introduce the notation $\models\varphi$.

Rule R1 preserves the validity of the corollary ψ: from $\models\varphi$ and $\models(\varphi \to \psi)$ it follows that $\models\psi$.

Rule R5, that is, replacement of equivalent formulas $\varphi(\chi) \leftrightarrow \psi, \chi \leftrightarrow \chi_1 \ \vdash\ \varphi(\chi_1) \leftrightarrow \psi$ preserves the validity of $\varphi(\chi_1) \leftrightarrow \psi$ after replacing some occurrences χ in φ(χ) by χ_1. By induction on the complexity of the formulas it can be shown that $V[\varphi(p_1,..., p_n] = \varphi(V[p_1],...,V[p_n])$, from where the validity of $\varphi(\chi_1) \leftrightarrow \psi$ follows by virtue of validity of $\varphi(\chi_1) \leftrightarrow \psi$ and $\chi \leftrightarrow \chi_1$.

We show that the rule R*2 also preserves the validity of formulas. To do this, it suffices to prove that the substitution of $p \vee q$, $p \,\&\, q$, in axioms, $ERA_{0.1}$ preserves their validity.

Remark 15-5. The substitution rule R*2 allows substituting into formulas φ only χ such that $\chi \in M^* = \{[p \,\&\, q,\ p \vee q\}]$. This means that the range of χ is HPW_h, where $HPW_h \in \overline{HPW}$, and the range of $\Box p$, $\Diamond p$ and $\urcorner p$ is a partition $\overline{HPW} =$, $\overline{HPW_v} \cup \overline{HPW_\tau} \cup \overline{IHPW}$ such that $\overline{HPW_v}$, $\overline{HPW_\tau}$, $\overline{IHPW}$ are the ranges for $\Box p$, $\Diamond p$ and $\urcorner p$, respectively, which correspond to empirical regularities ($\Box p$, $\Diamond p$) and their absence ($\urcorner p$). $\Box p$, $\Diamond p$ and $\urcorner p$ themselves cannot be substituted.

We consider (□2) $\Box p \to p$. R*2 $\int_p^{p\&q}$(□2) $\models \Box(p \,\&\, q) \to (p \,\&\, q)$, $\Box(p \,\&\, q) \leftrightarrow (\Box p \,\&\, \Box q)$, $(\Box p \,\&\, \Box q) \to (p \,\&\, q)$, since R5 and two-valued Propositional logic were used. Therefore, by virtue of the validity of $\Box p \to p$ and $\Box q \to q$, $\Box(p \,\&\, q) \to (p \,\&\, q)$ is valid.

The validity of (□4) $\Box(p \vee q) \to (p \vee q)$ is proved similarly, as well as the validity of $\Diamond(p \,\&\, q) \to (p \,\&\, q)$ and $\Diamond(p \vee q) \to (p \vee q)$ for (◇2).

We consider (□3) $\Box(p \,\&\, q) \leftrightarrow (\Box\mathrm{p} \& \Box\mathrm{q})$. $\int_p^{p\&q}$(□3) $\models \Box((p \,\&\, q) \,\&\, q) \leftrightarrow (\Box(p \,\&\, q) \,\&\, \Box q)$, $\Box((p \,\&\, q) \,\&\, q) \leftrightarrow$

$\Box(p \,\&\, q)$, $\Box(p \,\&\, q) \leftrightarrow (\Box p \,\&\, \Box q)$, but $(\Box(p \,\&\, q) \,\&\, \Box q) \leftrightarrow ((\Box p \,\&\, \Box q) \,\&\, \Box q)$, hence, $(\Box(p \,\&\, q) \,\&\, \Box q) \leftrightarrow \Box(p \,\&\, q)$ by virtue of R5, therefore, $\Box((p \,\&\, q) \,\&\, q) \leftrightarrow (\Box(p \,\&\, q) \,\&\, \Box q)$ is valid by virtue of the validity of $\Box(p \,\&\, q) \leftrightarrow (\Box p \,\&\, \Box q)$.

We consider $\int_p^{p\&q}(\Box 3) \models \Box((p \vee q) \,\&\, q) \leftrightarrow (\Box(p \vee q) \,\&\, \Box q)$, $\Box((p \vee q) \,\&\, q) \leftrightarrow \Box q$, as $(p \vee q) \,\&\, q) \leftrightarrow q$, and $(\Box(p \vee q) \,\&\, \Box q) \leftrightarrow (\Box p \vee \Box q) \,\&\, \Box q)$ by virtue of R5; but $((\Box p \vee \Box q) \,\&\, \Box q) \leftrightarrow \Box$q, therefore, $\Box((p \vee q) \,\&\, q) \leftrightarrow (\Box(p \vee q) \,\&\, \Box q)$ is valid.

The validity of the results of substitution $\int_p^{p\&q}(\Box 4)|$, $\int_p^{p\&q}(\Box 4)|$, as well as the validity of the substitution in $(\Diamond 3)$ and $(\Diamond 4)$ is proved similarly.

We consider $(\neg\,\Box)$ $\neg\,\Box \mathbf{p} \rightarrow (\Diamond p \vee \neg p)$, $\int_p^{p\&q}(\neg\,\Box) \models \neg\,\Box(p \,\&\, q) \rightarrow (\Diamond(p \,\&\, q) \vee \neg(p \,\&\, q))$.

$(\neg\,\Box p \rightarrow (\Diamond p \vee \neg p)) \leftrightarrow (\Box p \vee \Diamond p \vee \neg p)$, $p \leftrightarrow (\Box p \vee \Diamond p)$, as $\Box p \rightarrow p$, $\Diamond p \rightarrow p$, then $(\Box p \vee \Diamond p) \rightarrow p$; but by virtue of $\neg\,\Box p \rightarrow (\Diamond p \vee \neg p)$ we have $\neg p \vee \Box p \vee \Diamond p$, therefore, $p \rightarrow (\Box p \vee \Diamond p)$, hence, $p \leftrightarrow (\Box p \vee \Diamond p)$. $(\Box p \vee \Diamond p \vee \neg p) \leftrightarrow (\Box p \vee \Diamond p \vee (\neg\Box p \,\&\, \neg\Diamond p))$, $(\Box p \vee \Diamond p \vee (\neg\Box p \,\&\, \neg\Diamond p)) \leftrightarrow (\Box p \vee \neg\Box p \vee \Diamond p) \,\&\, (\Diamond p \vee \neg\Diamond p \vee \Box p)$, but $(\Box p \vee \neg\Box p) \leftrightarrow t$, $(\Diamond p \vee \neg\Diamond p) \leftrightarrow t$, therefore, $(\Box(p \,\&\, q) \vee \neg\Box(p \,\&\, q) \vee \Diamond(p \,\&\, q)) \,\&\, (\Diamond(p \,\&\, q) \vee \neg\Diamond(p \,\&\, q) \vee \Box(p \,\&\, q)) \leftrightarrow t$.

Hence, $\neg\,\Box(p \,\&\, q) \rightarrow (\Diamond(p \,\&\, q) \vee \neg(p \,\&\, q))$ is valid.

Similarly, we show that $\neg\,\Box(p \vee q) \rightarrow (\Diamond(p \vee q) \vee \neg(p \vee q))$ and the results of substitution of $p \,\&\, q$ and $p \vee q$ into $(\neg\Diamond)$ are valid.

We consider $(\Box\Box)$ $\Box\Box\, p \rightarrow \Box p$, show that $\int_p^{p\&q}(\Box\Box) \models \Box\Box(p \,\&\, q) \rightarrow \Box(p \,\&\, q)$ is a valid formula.

$\Box\Box(p \,\&\, q) \leftrightarrow \Box(\Box p \,\&\, \Box q)$ by virtue of $(\Box 3)$ and R5, $\Box(\Box p \,\&\, \Box q) \leftrightarrow (\Box\Box p \,\&\, \Box\Box q)$ by virtue of $(\Box 3)$ and R5; but $\Box\Box p \rightarrow \Box p$, $\Box\Box q \rightarrow \Box q$ according to $(\Box\Box)$, therefore, $\Box\Box(p \,\&\, q) \rightarrow \Box(p \,\&\, q)$ is valid.

We show that the result $\int_p^{p\&q}(\Box\Box) \models \Box\Box(p \vee q) \rightarrow \Box(p \vee q)$ is valid.

We consider $\Box\Box(p \vee q)$, $\Box\Box(p \vee q) \rightarrow \Box(\Box p \vee \Box q)$ by virtue of $(\Box 4)$ and R5; but $\Box(\Box p \vee \Box q) \leftrightarrow (\Box\Box p \vee \Box\Box q)$ also by virtue $(\Box 4)$ and R5. As $\Box\Box p \rightarrow \Box p$ and $\Box\Box q \rightarrow \Box q$ according to $(\Box\Box)$, then $(\Box\Box p \vee \Box\Box q) \rightarrow (\Box p \vee \Box q)$, $(\Box p \vee \Box q) \leftrightarrow \Box(p \vee q)$ $(\Box 4)$. Hence, $\Box\Box(p \vee q) \rightarrow \Box(p \vee q)$ is valid.

We consider $(\Box\Diamond)$. $\Box\Diamond p \rightarrow \Diamond p$; show that the result $\int_p^{p\&q}(\Box\Diamond) \models \Box\Diamond(p \,\&\, q) \rightarrow \Diamond(p \,\&\, q)$ is a valid formula. $\Box\Diamond(p \,\&\, q) \leftrightarrow \Box(\Diamond p \,\&\, \Diamond q)$ [$(\Diamond 3)$, R5], $\Box(\Diamond p \,\&\, \Diamond q) \leftrightarrow (\Box\Diamond p \,\&\, \Box\Diamond q)$[$(\Box 3)$, R5], $\Box\Diamond p \rightarrow \Diamond p$, $\Box\Diamond q \rightarrow \Diamond q$ [$(\Box\Diamond)$]; $(\Box\Diamond p \,\&\, \Box\Diamond q) \rightarrow (\Diamond p \,\&\, \Diamond q)$ by virtue of the two-valued Propositional logic, but $\Diamond(p \,\&\, q) \leftrightarrow (\Diamond p \,\&\, \Diamond q)$ [$(\Diamond 3)$], hence $\Box\Diamond(p \,\&\, q) \rightarrow \Diamond(p \,\&\, q)$ is a valid formula.

We also show that the result $\int_p^{p\&q}(\Box\Diamond) \models \Box\Diamond(p \vee q) \rightarrow \Diamond(p \vee q)$ is a valid formula. $\Box\Diamond(p \vee q) \leftrightarrow \Box(\Diamond p \vee \Diamond q)$ [$(\Diamond 4)$, R5], $\Box(\Diamond p \vee \Diamond q) \leftrightarrow (\Box\Diamond p \vee \Box\Diamond q)$[$(\Box 4)$, R5], $\Box\Diamond p \rightarrow \Diamond p$, $\Box\Diamond q \rightarrow \Diamond q$ [$(\Box\Diamond)$]; $(\Box\Diamond p \vee \Box\Diamond q) \rightarrow (\Diamond p \,\&\, \Diamond q)$ by virtue of the two-valued Propositional logic, but $\Diamond(p \vee q) \leftrightarrow (\Diamond p \vee \Diamond q)$ [$(\Diamond 3)$], hence $\Box\Diamond(p \vee q) \rightarrow \Diamond(p \vee q)$ is a valid formula.

Next, we show that the result $\int_p^{p\&q}(\Diamond\Box) \models \Diamond\Box(p \,\&\, q) \rightarrow \neg(p \,\&\, q)$ is a valid formula. We consider $\int_p^{p\&q}(\Diamond\Box) \models \Diamond\Box(p \,\&\, q) \rightarrow \neg(p \,\&\, q)$, $\Diamond\Box(p \,\&\, q) \rightarrow \neg(p \,\&\, q)$, but $\Diamond\Box(p \,\&\, q) \leftrightarrow \Diamond(\Box p \,\&\, \Box q)$ [$(\Box 3)$, R5]. We have $\Diamond\Box p \rightarrow \neg p$, $\Diamond\Box q \rightarrow \neg q$ [$(\Diamond\Box)$]; then $(\Diamond\Box p \,\&\, \Diamond\Box q) \rightarrow (\neg p \,\&\, \neg q)$ by virtue of the two-valued Propositional logic, but $(\neg p \,\&\, \neg q) \rightarrow (\neg p \vee \neg q)$, and $(\neg p \vee \neg q) \leftrightarrow \neg(p \,\&\, q)$, and therefore $\Diamond\Box(p \,\&\, q) \rightarrow \neg(p \,\&\, q)$ is a valid formula.

We consider $\int_p^{p\&q}(\Diamond\Box) \models \Diamond\Box(p \vee q) \rightarrow \neg(p \vee q)$.

According to the semantics of the histories of possible worlds, we have three possibilities for assessing $\Diamond\Box(p \vee q)$ according to the law of the excluded fourth.

(5) Cd $(\Diamond\,\Box\,(p \vee q)) = \upsilon...\upsilon$ is impossible since $\Diamond$ generates the end of the code Cd $\tau...\tau\;\upsilon...\upsilon$

(6) Cd $(\Diamond\,\Box\,(p \vee q)) = \tau...\tau\;\upsilon...\upsilon$ is impossible since the beginning of Cd is $\upsilon...\upsilon$

(7) Therefore, only the irregular code Cd is possible, and therefore we have $\neg(p \vee q)$.

Consequently, $\Diamond\Box(p \vee q)$ implies $\neg(p \vee q)$, and therefore $\Diamond\Box(p \vee q) \rightarrow \neg(p \vee q)$ is a valid formula.

The validity of $(\Diamond\Diamond)$ $\Diamond\Diamond p \rightarrow \neg p$ is established similarly.

We consider $(\Box \,\&\, \Diamond)$, $\neg(\Box p \,\&\, \Diamond p)$ and $\int_p^{p\&q}(\Box \,\&\, \Diamond) \models \neg(\Box(p \,\&\, q) \,\&\, \Diamond(p \,\&\, q))$, $\Box(p \,\&\, q) \,\&\, \Diamond(p \,\&\, q) \leftrightarrow (\Box p \,\&\, \Box q) \,\&\, (\Diamond p \,\&\, \Diamond q) \leftrightarrow (\Box p \,\&\, \Diamond p) \,\&\, (\Box q \,\&\, \Diamond q) \leftrightarrow f \,\&\, f \leftrightarrow f$, as $(\Box p \,\&\, \Diamond p) \leftrightarrow f$, $(\Box q \,\&\, \Diamond q) \leftrightarrow f$, hence $\neg(\Box(p \,\&\, q) \,\&\, \Diamond(p \,\&\, q)) \leftrightarrow \neg f$, $\neg f \leftrightarrow t$.

In addition to syntactic proof of validity of $\int_p^{p\&q}(\Box \,\&\, \Diamond)\,|$, a simple proof of the validity of this formula is possible by establishing the inconsistencies of $Cd(\Box p)$ and $Cd(\Diamond p)$ using the semantics of the history of possible worlds.

We consider $\int_p^{p\vee q}(\Box \,\&\, \Diamond) \models \neg(\Box(p \vee q) \,\&\, \Diamond(p \vee q))$, $Cd\Box(p \vee q) = v...v$, $Cd\Diamond(p \vee q) = \tau...\tau v...v$, hence, $(\Box(p \vee q) \,\&\, \Diamond(p \vee q)) \leftrightarrow f$ and $\neg(\Box(p \vee q) \,\&\, \Diamond(p \vee q)) \leftrightarrow t$.

We establish the validity of the formulas $(\Box\ \&\ \neg)$ and $(\Diamond\ \&\ \neg)$. $(\Box\ \&\ \neg)$: $\neg(\Box p\ \&\ \neg p)$, consider the formula $p \leftrightarrow (\Box p \vee \Diamond p)$, it is obvious that it is provable in $ERA_{0.1}$. Its provability follows from $(\Box 2)$ $\Box p \rightarrow p$; $(\Diamond 2)$ $\Diamond p \rightarrow p$ and $(\neg\Box)$ $\neg\Box p \rightarrow (\Diamond p \vee \neg p)$, since $\vdash ((\Box p \vee \Diamond p) \rightarrow p)$ and $\vdash (\neg p \vee \Box p \vee \Diamond p)$ then $\vdash (p \leftrightarrow (\Box p \vee \Diamond p))$. From $\vdash (p \leftrightarrow (\Box p \vee \Diamond p))$ we obtain $\vdash (\neg p \leftrightarrow (\neg\Box p\ \&\ \neg\Diamond p))$. Applying R5 to $(\Box\ \&\ \neg)$ and $(\neg p \leftrightarrow (\neg\Box p\ \&\ \neg\Diamond p))$, we obtain $\neg(\Box p\ \&\ (\neg\Box p\ \&\ \neg\Diamond p)) \leftrightarrow \neg f$. It follows that $\int_p^{p\vee q}(\Box\ \&\ \neg)|$ and similarly $\int_p^{p\vee q}(\Box\ \&\ \neg)|$ are valid.

As $\Box\neg p \leftrightarrow f$ and $\Diamond\neg p \leftrightarrow f$, then validity of $\int_p^{p\& q}(\Box\neg)|$, $\int_p^{p\vee q}(\Box\neg)|$ and $\int_p^{p\& q}(\Diamond\neg)|$, $\int_p^{p\vee q}(\Box\neg)|$ holds for the results of substitutions in $\neg\Box\neg p$ and $\neg\Diamond\neg p$, respectively.

The validity of the results of substituting $p\ \&\ q$ and $p \vee q$ into the axioms of $\mathrm{ERA}_{0.1}$ is an induction basis for the complexity of formulas φ such that $\varphi \in M^* = [\{p \vee q, p\ \&\ q\}]$, which proves the correctness of the substitution rule R*2.

The inference rules R3 $\Box\varphi, \Box(\varphi \rightarrow \psi) \vdash \Box\psi$ R4 $\Diamond\varphi, \Diamond(\varphi \rightarrow \psi) \vdash \Diamond\psi$ also preserve the validity of the consequences if the parcels are valid, which follows from the definition of the assessment function for $\Box$, $\Diamond$ and $\rightarrow$.

Lemma 2-5 is proved.

Lemma 1-5 and Lemma 2-5 imply

Proposal 5-5. The $\mathrm{ERA}_{0.1}$ calculus is consistent.

We give some theorems of $\mathrm{ERA}_{0.1}$:

1. $\Box p \vee \Diamond p \vee \neg p$

2. $p \leftrightarrow (\Box p \vee \Diamond p)$

3. $(\Box p\ \&\ q) \leftrightarrow (\Box p\ \&\ \Box q) \vee (\Box p\ \&\ \Diamond q)$

4. $(\Diamond p\ \&\ q) \leftrightarrow (\Diamond p\ \&\ \Diamond q) \vee (\Diamond p\ \&\ \Box q)$

5. $(\Box p \vee q) \leftrightarrow (\Box p \vee \Box q \vee \Diamond q)$

6. $(\Diamond p \vee q) \leftrightarrow (\Diamond p \vee \Box q \vee \Diamond q)$

7. $p \vee \neg p$

Remark 16-5. Some amplifications to $ERA_{0.1}$ are possible by adding the axioms $(\Box\Box_1)$ $\Box p \rightarrow \Box\Box p$ and $(\Diamond)$ $\Diamond p \rightarrow \Box\Diamond p$, which belong, respectively, to the modal logics S4 and S5 [40, 41].

It can be shown that $(\Box\Box_1)$ and $(\Diamond)$ are valid in the semantics of a finite set of histories $\overline{HPW}$ of finite possible worlds, and also that for the corresponding extensions $ERA_{0.1}$ $ERA_{0.1.4} = ERA_{0.1}$ and $\Box p \rightarrow \Box\Box p$, $ERA_{0.1.5} = ERA_{0.1}$ and $\Diamond p \rightarrow \Box\Diamond p$, $ERA_{0.1.4.5} = ERA_{0.1}$ and $\Box p \rightarrow \Box\Box p$, $\Diamond p \rightarrow \Box\Diamond p$ there is an analogue of Proposal 5-5: these calculi are consistent.

The $ERA_{0.1}$ logic only partially imitates the JSM research by propositional means in accordance with the semantics of possible worlds from $\overline{HPW}$, using the operators $\Box$ and $\Diamond$ that correspond to empirical regularities, that is, empirical laws and trends. $ERA_{0.1}$ does not represent abduction of the second kind, which is formulated taking into account two theories of truth [24]: correspondent [25, 26] and coherent [23].

In order to display and simulate propositional methods, JSM research completed with abductive inference of the second kind, we extend the $ERA_{0.1}$ logic using the TM fragment formulated below, which uses the operator T: "it is true that...." The operator T was introduced by G.H. von Wright for the three logics of truth, that is, TL, $T'L$ and $T''L$, respectively [42]. The proposed TM fragment containing the operator T extends the logic $ERA_{0.1}$ to ERA_1, where ERA_1 is $ERA_{0.1}$ with the addition of TM, formulated below.

ERA_1 logic.

Alphabet: $p, q, r, \ldots, 1, \&, \vee, \rightarrow, \Box, \Diamond, T, (,)$.

Formula definition:

1) p, q, r are formulas;

2) if φ is a formula, then 1φ is a formula;

3) if φ, ψ are formulas, then $(\varphi\ \&\ \psi)$, $(\varphi \vee \psi)$, $(\varphi \rightarrow \psi)$ are formulas;

4) if φ is a formula, then $\Box\varphi$, $\Diamond\varphi$ are formulas;

5) if φ is a formula, then $T\varphi$ is a formula;

6) there are no other formulas.

There are axioms and inference rules of $\mathrm{ERA}_{0.1}$.

Additional rule for TM fragment: $\mathrm{R_T1}$. $\mathrm{T}\varphi$, $\mathrm{T}(\varphi \rightarrow \psi) \vdash \mathrm{T}\psi$.

TM fragment.

Axioms $T1$–$T12$:

A1. $Tp \vee \neg Tp$

A2. $T\neg p \vee \neg T\neg p$

A3. $Tp \rightarrow p$

A4. $T\neg p \rightarrow \neg p$

A5. $T(p^{\sigma 1}\ \&\ q^{\sigma 2}) \leftrightarrow (Tp^{\sigma 1}\ \&\ Tq^{\sigma 2})$, where $p^{\sigma} = \begin{cases} p, \text{if } \sigma = 1 \\ \neg p, \text{if } \sigma = 0 \end{cases}$;

A6. $T(p^{\sigma 1} \vee q^{\sigma 2}) \leftrightarrow (Tp^{\sigma 1} \vee Tq^{\sigma 2})$

A7. $T\Box p \leftrightarrow (\Box p\ \&\ Tp)$

A8. $T\Diamond p \leftrightarrow (\Diamond p\ \&\ Tp)$

A9. $(Tp \rightarrow Tq) \rightarrow T(p \rightarrow q)$

A10. $((\Box(p \rightarrow q)\ \&\ Tq) \rightarrow \Box p)$

A11. $((\Diamond(p \rightarrow q)\ \&\ Tq) \rightarrow \Diamond p)$

Derived inference rule: $\Box(p \rightarrow q), Tq \vdash \Box p$.

Obviously, the axioms A10 and A11 are important, representing the principle of abductive inference of the second kind. They correspond to the generation of empirical regularities, that is, empirical laws (with $\Box$) and empirical tendencies (with $\Diamond$).

The ERA_1 semantics is formed by the set $\overline{HPW}$ and a set **Tr**, where **Tr** is an open set of true formulas Tp and $T\neg p$.

The set $\overline{HPW}$ is used to determine the assessment function $V[\varphi]$ for a coherent theory of truth [23], and the set **Tr** is used to determine $V[\varphi]$ for the correspondent theory of truth [25, 26]. $V[Tp] = t$, if and only if $Tp \in Tr$.

TM is consistent with respect to semantics with $\overline{HPW}$ and **Tr**.

The following ERA_1 theorems are obvious: $\neg p \to \neg Tp$, $p \to \neg T\neg p$, $T\Box p \to p$, $\neg p \to \neg T\Box p$, $T\Diamond p \to p$, $\neg p \to \neg T\Diamond p$.

Using the derived inference rule $\Box(p \to q)$, $Tq \vdash \Box p$ and the deduction theorem, we obtain the statement $Tq \vdash \Box(p \to q) \to (\Box p \to \Box q)$.

We note that the modal system of G.H. von Wright M [40, 41, 43] contains the axioms $\Box p \to p$ and $\Box(p \to q) \to (\Box p \to \Box q)$ and the inference rule $\varphi \vdash \Box\varphi$, where φ is the tautology of two-valued logic that does not hold in ERA_1.

ERA_1 with modal operators $\Box$ and $\Diamond$ corresponds to the Tree T_1, which represents simplified empirical regularities. The Tree T_2 represents a variant of modal logic of the ERA type with modal operators $\Box$, $\Diamond$, and ∇, where ∇ is the weak possibility operator corresponding to the condition $2q \geq s + 1$ for codes of empirical tendencies.

CONCLUDING REMARKS

The JSM method of automated research support is an artificial intelligence method, which is an area of research such that their objective is to imitate and enhance the cognitive process and rational human behavior through computer systems. Therefore, clarification of the terms "method" and "research," "cognitive process," "computer system" is needed.

By **method** we mean a set of principles and procedures such that their application forms a **research**, the result of which is to obtain new knowledge used in the formation of an open theory (quasi-axiomatic theory according to the ASSR JSM method).

By **research** we mean the solution of problems using a **method** expressed in a language that has **descriptive** and **argumentative** functions [44], such that the application of the method generates **empirical regularities**, and its results concede **falsification** [44].

By **cognitive process** we mean the process of knowledge discovery such that it is formed by the **analysis** of data (facts), **prediction** and **acceptance** of research results using the explanation of these results.

By **computer system**, which is a product of artificial intelligence, we mean: (1) software systems performing some procedures from the arsenal of AI (for example, decision trees, neural networks, etc.); we will call them **artificial intelligence systems**; (2) **intelligent systems** (ISs) that have the following architecture: fact bases and knowledge bases, a Problem Solver and a comfortable interface, where a Problem Solver = Reasoner + Calculator + Synthesizer, moreover, IS performs basic intellectual abilities [11] (including: recognition of essential parameters in the data, reasoning and synthesis of cognitive procedures, argumentation, training, reflection, integration of knowledge, etc.).

Operation of intelligent systems goes in two modes, that is, automatic and interactive, which perform an intelligent process formed by imitating and enhancing the thought process and supporting the cognitive process, which requires expanding the facts bases FB(p) and finding stable regularities in them, that is, empirical regularities (sets $ER = EL \cup ET \cup WET$ in the ASSR JSM method). Rational behavior is imitated through the logic of argumentation [29].

Finally, once again, by method we mean the organization of concepts, principles and procedures, the use of which is a means of obtaining **new knowledge** including empirical regularities.

The method is both a means of forming a theory and its implementation; it may contain heuristics [1, 2], which apply plausible reasoning to the initial data. The described **idea** of the method involves the use of empirical data, and, therefore, refers to **open** theories.

The idea of "knowledge in a computer system" is defined as follows:

(1) Zero-level knowledge (Knowledge_0): elements of the facts base FB(p), where the fact is represented by elementary statements with attributed truth values ("1" is true, "−1" is false, and "τ" is uncertain);

(2) First-level knowledge (Knowledge_1): logical combinations of knowledge of the zero level;

(3) Second-level Knowledge (Knowledge_2): representation of procedures (procedural knowledge) and hypotheses obtained by applying procedures;

(4) Third-level knowledge (Knowledge_3): axioms of quasi-axiomatic theories (QAT) are descriptive axioms and axioms of data structure;

(5) Fourth-level knowledge (Knowledge_4), discovered empirical regularities corresponding to the intensional ER and represented by its extensional, forming some forest through a set of strategies $\overline{Str}$ and $\overline{ICF}$ are sets of integral causal forcings that generate empirical nomological statements (ENS) $A^{\sigma}_{\chi}(C', Q)$, where $\chi \in E = \{a, b, ..., m, n\}$, and $\sigma = +, -$.

We note that IS performs an intellectual process, which is the interaction of imitation and amplification of the thought process, formalized by the synthesis of cognitive procedures (induction + analogy + abduction, i.e., *JSM* reasoning), and the cognitive process of detecting empirical regularities corresponding to ER, which means knowledge discovery, replenishing the knowledge base of intelligent systems.

Thus, the JSM method is an automated research support using **heuristics** [1, 2] formed by JSM reasoning and ENS detection procedures. Let us note to the principle of accepting the results of the JSM research using two scales for assessing the quality of reasoning and hypotheses, using which the **nondeterioration** of the characteristics presented in these scales is controlled with a continuous expansion of the fact bases [6].

In the *Appendices*, we clarify and supplement the statements of this conclusion.

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APPENDIX

1. Abduction of the second kind can be formulated in the following equivalent way using parcels (1), (2) and (3):

(1) $A_\chi^\sigma(C', Q)$, where $\chi \in E$;

(2) $M\chi\, \forall Z \forall p \forall h \exists n\, (J_{\langle 1,n\rangle}H_2(C', Q, p, h)$
$\to J_{\langle 1,n+1\rangle}H_1(Z, Q, p, h))$;

(3) $\forall Z((C' \subset Z) \to \mathrm{Ver}[J_{\langle 1,\bar{n}+1\rangle}H_1(Z, Q, \bar{s}, \bar{h})] = t)$;

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(4) $M\chi\, \forall p \forall h \exists n\, J_{\langle 1,n\rangle}H_2(C', Q, p, h)$,

where M_χ is $\Box_\chi$, $\Diamond_\chi$ and ∇_χ, and $\chi \in E = \{a, b, \ldots, m, n\}$.

We note that parcels (1) and (2) have truth values according to the coherent theory of truth, and parcel (3) uses the correspondent theory of truth. Therefore, corollary (4) is obtained according to the interaction of two theories of truth.

We also note that (2) is a consequence of (1), and the ERA_1 derived rule is $M_\chi(p \to q)$, $Tq \vdash M_\chi p$ is a propositional imitation of abduction of the second kind.

2. In [1, 2], the principle of the modal trace $M_1M_2\ldots M_k$ was formulated, generated by the continuation of the sequence of nested FB(p) and the formation of the corresponding sequence of histories of possible worlds $\overline{HPW}_1, \overline{HPW}_2, \ldots, \overline{HPW}_k$, which correspond to modalities $M_1, M_2, \ldots, M_k$.

Since the modal operators M_χ corresponding to the Tree T and the set of integral causal forcings $\overline{ICF}$ are partially ordered, then the sequence $M_1, M_2, \ldots, M_k$ will be called regular if $M_1 \sqsubseteq M_2 \sqsubseteq \ldots \sqsubseteq M_{k-1} \sqsubseteq M_k$.

The sequences of M_χ-operators corresponding to $Str_{x,y}$ will be denoted by $\tilde{\mathbf{M}}(x, y)$. Obviously, the set of all $\tilde{\mathbf{M}}(x, y)$, corresponding to the set $\overline{Str}$ of all strategies of *JSM* reasoning $Str_{x,y}$ [13], can be ordered as follows: $\tilde{\mathbf{M}}_1(x_1, y_1) \sqsupseteq \tilde{\mathbf{M}}_2(x_2, y_2)$, if and only if $M_i^{(1)} \sqsupseteq M_i^{(2)}$ for $i = 1, \ldots, k$ and $\langle x_1, y_1\rangle \geq \langle x_2, y_2\rangle$ [13], where $M_i^{(1)}$ and $M_i^{(2)}$ are modal sequence operators $\tilde{\mathbf{M}}_1(x_1, y_1)$ and $\tilde{\mathbf{M}}_2(x_2, y_2)$, respectively.

Let M be the set of all sequences of M_χ-operators; then in M there exist the largest and the smallest elements.

We now state the principle of a **successful** modal trace:

the modal trace is **successful** for k-histories of possible worlds HPW that are sequentially expandable and generate $\overline{HPW}$, if there is a strategy of *JSM* reasoning $Str_{x,y}$ such that the corresponding sequence $\tilde{\mathbf{M}}(x, y)$ obtained by an acceptable *JSM* research according to the definition *Df.*20-4.

A propositional imitation of a successful JSM research is the nonfinite S4 and S5 similar ERA_1 amplifications by adding the axioms $\Box p \to \Box\Box\ldots\Box_k p$ and $\Diamond p \to \Box\Box\ldots\Box_k \Diamond p$ for all k that correspond to regular Cd codes of empirical regularities.

3. We now state the conditions for an **ideal** JSM research.

(1) There exists $Str_{x,y}$ such that the condition holds: if $\Omega(p) \subseteq \Omega(q)$, then $\bar{O}_{x,y}(\Omega(p)) \subseteq \bar{O}_{x,y}(\Omega(q))$. Then, the *JSM* operator $\bar{O}_{x,y}(\Omega(p))$ is a closure.

(2) For $Str_{x,y}$, satisfying Condition (1), the following statement holds: for any $\langle V, Y\rangle$ and all p, h if $J_{\langle 1,n\rangle}H_2(V, Y, p, h) \vee J_{\langle -1,n\rangle}H_2(V, Y, p, h)$ holds, then $\langle V, Y\rangle \in \mathrm{G}_{x,y} = (\bigcup_{\chi\in E}\{\langle V, Y\rangle | A_\chi^+(V, Y)\}) \cup (\bigcup_{\chi\in E}\{\langle V, Y\rangle | A_\chi^-(V, Y)\})$, where $A_\chi^\sigma(C', Q)$ are realization *ICF* for $\langle C', Q\rangle$, $\chi \in E$, and $\neg(\mathrm{G}_{x,y} = \Lambda)$.

(3) For $Str_{x,y}$, satisfying Condition (1), the causal completeness axioms $\mathrm{CCA}^{(\sigma)}$ are true, where $\sigma \in +, -$ [6].

(4) $\bar{O}_{x,y}(\Omega(s))| \geq |\Omega^\tau(0)|$ and $m_0 = l_0$, where s is the number of the last expansion of FB(p), $m_0 = |\Omega\tau(0)|$, and l_0 is the number of correct predictions of the studied effect Q.

(5) For $Str_{x,y}$ satisfying Condition (1), there exists a successful sequence $\tilde{\mathbf{M}}(x, y)$ such that $k \geq 3$ (k successful $\tilde{\mathbf{M}}(x, y)$).

(6) Complete JSM research for all $Str_{x,y}$ from a given set $\overline{Str}$ is characterized by the following scheme.

1^0. $\Omega(0, 1), \Omega(1, 1), \ldots, \Omega(s, 1)$; $\Omega(0, 1) \subset \Omega(1, 1) \subset \ldots \subset \Omega(s, 1)$,

2^0. $\Omega^\tau(0, 1)$, $\Omega^\tau(0, 1) = \Omega^\tau(p, h)$ for all p and h, where $0 \leq p \leq s$, $h \in \overline{HPW}$;

3^0. $\overline{HPW}$, $|\overline{HPW}| = (s + 1)!$;

4^0. $\overline{Str}$,

5^0. $\overline{ICF}$,

6^0. $\overline{6^0.\Im,}$,

where $\Im = \{[\Im_{x,y}]_E | (x \in I^+) \& (y \in I^-)\}$, $[\Im_{x,y}]_E = \langle \Sigma \cup \Sigma_E, \tilde{\Omega}_{x,y}(\overline{s}, (\overline{s}+1)!) \cup \tilde{\Delta}(\overline{s}, (\overline{s}+1)!, R) \rangle$, Σ_E, many of all $A_\chi^\sigma(C', Q)$, corresponding to $G_{x,y}$, where $\chi \in E$.

We suppose that there exists a QAT such that for $Str_{x,y}$ Conditions (1)–(6) are satisfied.

The following condition holds: $\Im$ belong to $\left[\Im_{\langle (ad_0b)^+, \neg a^- \rangle}\right]_E$ and $\left[\Im_{\langle \neg a^+, (ad_0b)^- \rangle}\right]_E$, where $\langle (ad_0b)^+, \neg a^- \rangle$ and $\langle \neg a^+, (ad_0b)^- \rangle$ are the largest elements of direct products of lattices $Int(L^+x\neg L^-)$ and $Int(\neg L^+xL^-)$ for inductive inference rules (I^+) and (I^-), respectively [13]. Wherein $A_a^+\left(C_1', Q_1\right)$ and $A_a^-\left(C_2', Q_2\right)$ correspond to $\Im_{\langle (ad_0b)^+, \neg a^- \rangle}$ and $\Im_{\langle \neg a^+, (ad_0b)^- \rangle}$, where a is the index A_a^+ and A_a^- is the largest element of the partially ordered sets E^+ and E^-, respectively, where $E = E^+ \cup E^-$.

Conditions (1)–(6) have various attenuations that characterize real *JSM* research, which correspond to *ExtER* and a specific forest generated by this complete *JSM* research for $\overline{Str}$.

It is important to note that Σ_E contains empirical nomological statements (*ENS*) of three types of modalities $\Box_\chi$, $\Diamond_\chi$ and ∇_χ, which in a sense expresses the degree of nomology while maintaining universality using quantifiers $\forall Z \forall p$. *ENS* express the knowledge discovery, which is the goal of **data mining** as a means of **research** support and the formation of open theories (by virtue of this, ***open data*** is **more important** than ***big data***).

Translated by S. Avodkova

SPELL: OK